Mathematica Centrum

Together, let's shape the mathematicians of the future

PYTHAGORAS PREPARATORY TEST 2012
DETAILED SOLUTIONS

1. The number of edges of a cube (12) multiplied by the number of faces of a cube (6) is equal to 72.

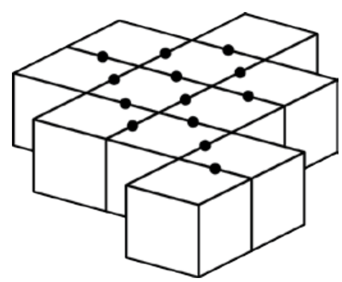
2. The factors of 6 are (1, 2, 3, and 6), those of 15 are (1, 3, 5, and 15). These 2 numbers have 2 factors in common.

3. The largest 4-digit even number that can be written with the digits 1, 8, 6, and 4 is 8 614.

4. The average of 0, 2, 4, 6, and 8 is ((0 + 2 + 4 + 6 + 8) ÷ 5) 4. In this case, it's the central term because the 5 numbers are evenly distributed.

5. The missing number in the sequence: 3 500, 3 250, ?, 2 750, 2 500 is (3 250 - 250) 3 000.

6. Each dot in the diagram accounts for 2 glued faces. There are (13 x 2) 26 glued faces. The number of faces that have no glue on them is (66 - 26) 40.



7. 16 quarters = 400¢ = 40 dimes

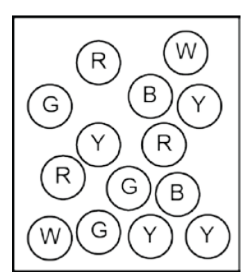
8. Write the following 5 numbers: 3 782, 2 863, 1 935, 2 926, 3 931 in increasing order (from the smallest to the largest). The fourth number written is 3 782.

9. The number that is 10 times smaller than 10 is 1. The number that is 10 more than 1 is 11.

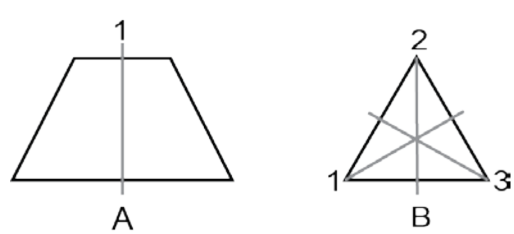
10. 11 hundreds - 280 + 14 tens = 1 100 - 280 + 140 = 960.

11. The base of a prism has 7 sides. The sum of the number of edges (7 + 7 + 7) plus the number of vertices (7 + 7) is 35.

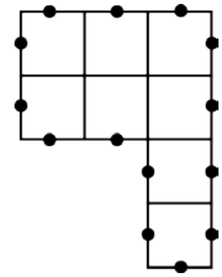
12. Without looking, Mathew picks one marble from the box. In this box there are 3 red, 3 green, 4 yellow, 2 black and 2 white marbles. Because there are 4 yellow marbles, the most prevalent colour in the box, Mathew is more likely to choose a marble of this colour.



13. The sum of the number of lines of symmetry of figure A (1) and of figure B (3) is equal to 4.



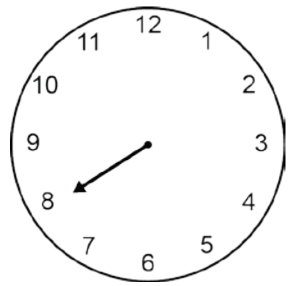
14. Since the digit 0 cannot be written first, we can only form (102, 120, 201, 210) four 3-digit numbers.



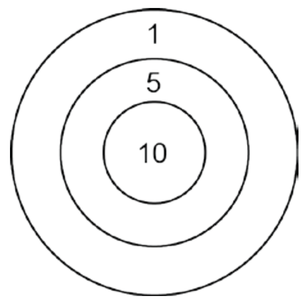
15. The perimeter of this figure is 14 cm.

16. Since Mathusalem hit the target 8 times and scored 38 points, we conclude that he hit the 1-point area 3 times. The only way to score the other 35 points is to hit the 10-point area twice and the 5-point area 3 times.

17. The clock shown in the diagram has lost its minute hand at approximately 7:55.



18. Instead of trying to answer this question right away, first let us analyse a simpler form of the same problem. How many odd numbers are there between 2 and 4? Evidently, there is only one, the 3. How many odd numbers are there between 2 and 10? There are (3, 5, 7, 9) 4. How many odd numbers are there between 8 and 18? There are (9, 11,13, 15, 17) 5. You notice that the number of odd numbers between 2 even numbers is always equal to half the difference between the even numbers. We can apply this rule to the original question. The number of odd numbers between 80 and 180 is ((180 - 80) ÷ 2) 50. Many great scientific and mathematical discoveries were made this way, by transforming the original problem into simpler models. From these we can more readily draw the mathematical law that can be used to solve all problems of the same type.



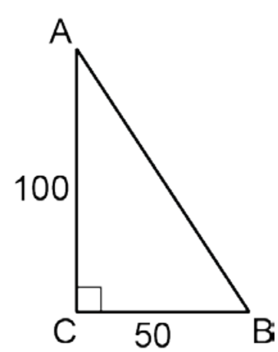
19. The purchase of III is better than that of I because he has bought double the amount of soap that I bought, but at a price which is (2 x \$4.50 = \$9.00) 10¢ less than double the price. The purchase of II is better than that of I because he has bought triple the amount of soap, but at a price that is (3 x \$4.50 = \$13.50) \$1.15 less than triple the price. We can thus conclude that the best buy was made by II only.

20. Any number whose digits add up to a sum which is divisible by 3 is a multiple of 3. The numbers 102 and 120 are thus multiples of 3. There are (105, 108, … 117) 5 multiples of 3 between 102 and 120. We can find this result by subtracting 1 from one third the difference between 120 and 102 (120 - 102 = 18, 18 ÷ 3 = 6 and 6 - 1 = 5).

21. 10% of \$80 = 10% x \$80 = 0.1 x \$80 = \$8.

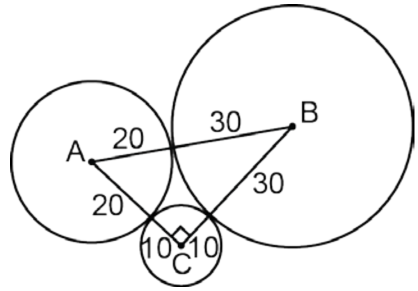
22. The fraction 8/15 < 3/5 because 3/5 = 9/15. The fraction 8/15 < 17/30 because 8/15 = 16/30. The fraction 1/2 or 6/12 < 7/12. The fraction 1/2 < 8/15 because 1/2 = 15/30 and 8/15 = 16/30.

23. The distance between them after 2 1/2 hours will be (2.5 x (16 - 12)) 10 km.



24. The area of the triangle is given by the formula B x H / 2. This area is equal to (50 x 100 ÷ 2) 2 500 cm^2.

25. The perimeter of triangle ABC is equal to the sum of AB + BC + CA = 50 cm + 40 cm + 30 cm which is 120 cm.

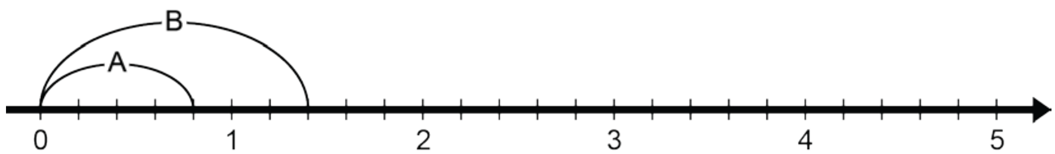


26. There are (11, 13, 17, 19, 23, and 29) 6 prime numbers between 10 and 30.

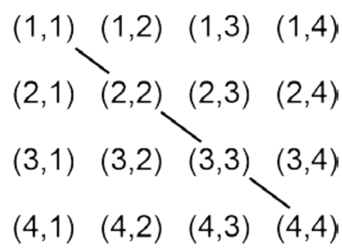
27. The difference between 1 million and 100 000 is (1 000 000 - 100 000) 900 000.

28. These two numbers could be 30 and 32. The new average is ((30 + 6) + (32 - 4) ÷ 2) 32. We can find the new average by adding ((6 - 4) ÷ 2)1 to 31, which gives 32.

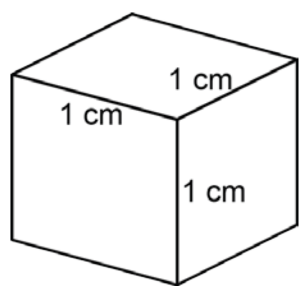
29. Spider A makes 5 jumps. It covers a distance of (5 x 4/5) 4. Spider B makes 3 jumps. It covers a distance of (3 x 7/5) 21/5 or 4 1/5. The distance between the two spiders after A has made 5 jumps and B has made 3 jumps is (4 1/5 - 4) 1/5.



30. There are 16 possible outcomes. The pairs which lie on the diagonal of the table of possible outcomes are those where the first and second numbers are the same. The pairs above the diagonal are those where the first number is smaller than the second. The pairs that are under this diagonal are those where the second number is smaller than the first. The probability that the outcome is one of these pairs is (6/16) 3/8.



31. The area of one face is (1 cm x 1 cm) 1 cm^2. The area of the 6 faces of the cube is 6 cm^2. The cube's volume is (1 cm x 1cm x 1cm) 1 cm^3. The ratio of the area of the cube's faces compared to the cube's volume is equal to (6 cm^2 ÷ 1 cm^3) 6 cm^2 per cm^3?



32. The factors of 100 are {1, 2, 4, 5, 10, 20, 25, 50, 100}
The product of all the factors of 100 is equal to
(1 x 2 x 4 x 5 x 10 x 20 x 25 x 50 x 100) =
(<u>1 x 100</u> x <u>2 x 50</u> x <u>4 x 25</u> x <u>5 x 20</u> x 10) =
(<u>100</u> x <u>100</u> x <u>100</u> x <u>100</u> x 10) = 10^9.

33. Two (2) objects, A and B, have different weights. The weight of each one in pounds is a positive integer. Let A represent the weight of object A and B represent the weight of object B. We know that 8**A + **3**B =** 68 and 6**A + **9**B =** 78. What is the weight in pounds of 3**A** + 5**B**? This problem is about diophantine equations, equations where the values of the unknowns (the letters A and B) are <u>integers</u>. In this case, these integers are in reality the weights of the two objects. Our objective is to find the weight of each object. There are two clues (2 equations) to help us find the weight of each one: eight times object A (which is written mathematically 8A) and 3 times object B weigh a total of 68 pounds, six times objet A + nine times object B weigh a total of 78 pounds. Once we find the weight of each object, we will be able to answer the question stated above: what is the weight in pounds of 3**A + **5**B**? Each letter can only take one value, the same, not only in the two equations, but also in the expression 3A + 5B. To solve this kind of problem, we must first start by trial and error, then proceed by observation and logic. First, let us look at equation: 8**A +** 3**B =** 68. Let us suppose that A = 1 (1 pound); this equation becomes 8 + 3B = 68, then it becomes 3B = 60 and we find that B = 20. Let us represent this possibility by the pair (**1, 20**). If A = 2, we find that 3B = 52 and B = 17 1/3. We reject this possibility because the value of B is not an integer. The next value of A for which B is an integer is A = 4. When A = 4, B = 12. The pair (**4,12**) is another possibility of the weights of A and B. The next one is (**7,4**). This outcome is also the last one, because B cannot take the value of -4 (You probably noticed that when the value of A increases by **3** (the number in front of the letter B), that of B decreases by **8** (the number in front of letter A). We find 3 possible values for A and B when we analyse the equation 8A + 3B = 68. Since we are looking for the weights of objects A and B (and keeping in mind that these weights must be the same for both equations) let's see which one of these 3 pairs is also a possibility for the equation 6**A +** 9**B =** 78. This pair is (**7,4**). In effect 6 x **7** + 9 x **4** = 78. The weight of object A is 7 pounds and that of B is 4 pounds. The weight of 3**A** + 5**B** is 3 x **7** + 5 x **4** = 41 pounds. You could have solved this problem by first analysing the equation 6A + 9B = 78. You would have found the exact same answer.