Mathematica Centrum

Together, let's shape the mathematicians of the future

EULER PREPARATORY TEST 2012
DETAILED SOLUTIONS

1. The value of (7 + 3) - (-8 + 2) is 10 - (-6) = 16.

2. -5 x 2 - (-5) = -10 + 5 = -5

3. The closest integer to the value of -3/4 x 6/12 + 3/8 is -3/4 x 1/2 + 3/8 = -3/8 + 3/8 = 0.

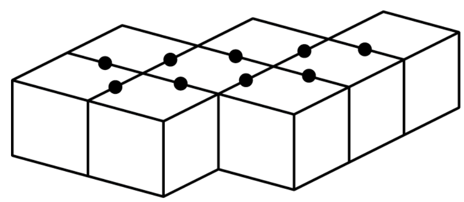
4. Thirty is (30/45 = 2/3) 2/3 of 45.

5. The sum of all the factors of 30 (1 + 2 + 3 + 5 + 6 + 10 + 15 + 30) is 72.

6. The result of 5/4 of 20% of 0.2 is equal to 5/4 x 1/5 x 1/5 = 1/20 = 5%.

7. The ratio of 0.08 to 0.2 is equal to 8/20 = 2/5. This ratio is the same as the ratio of 10 to 25.

8. Each dot in the diagram accounts for 2 glued faces. In all, there are (9 x 2) 18 glued faces. The number of faces that have no glue on them is (8 x 6 = 48 and 48 - 18) 30.

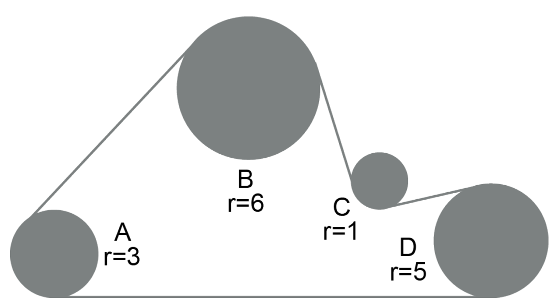


9. The value of N in the equation:
9 x 8 x 7 x 6 = 18 x N x 8 x 21 is 1.

10. Under the translation t: (-1, 6), the coordinates of point A (-1, 5) become A'(-2, 11), those of B (4, -2) become B'(3, 4) and those of C (-6, -1) become C'(-7, 5).

11. The measures of the acute angles in a right triangle are in the ratio 2:3. Since the sum of the two acute angles is 90°, we know that the smaller one is equal to 2/5 of 90° and the larger one is equal to 3/5 of 90°. The smaller one is equal to (2/5 x 90) 36°. Three times the value of the smaller one is equal to (3 x 36) 108°.

12. Mathew did 1/3 of the distance by car, then (3/4 x 2/3) 1/2 of the remaining distance by bus and (6/6 - 5/6 (1/3 + 1/2)) 1/6 of the total distance on foot.



13. If wheel B turns at 20 revolutions per minute, wheel C turns at (20 x 6) 120 revolutions per minute and wheel D turns at (120 ÷ 5) 24 revolutions per minute.

14. There are four 2-digit numbers less than 50 that have digits which add up to 5. These numbers are 14, 23, 32, and 41. Of these, only 23 and 41 are prime numbers.

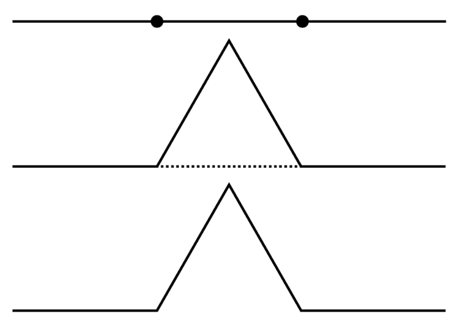
15. The rule of the sequence is - 5 x 5 + 5 ÷ 5 … . The next number in the sequence is (3 x 5) 15.

16. The average of -1/3 and 2/3 is equal to ((-1/3 + 2/3) ÷ 2) 1/6.

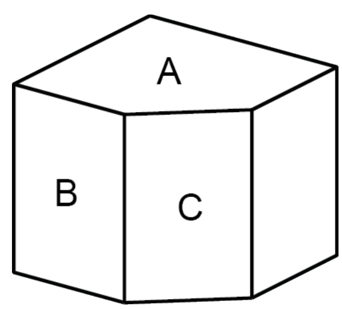
17. There are 4 positive integers less than 125 (1^3 = 1, 2^3 = 8, 3^3 = 27 and 4^3 = 64) that are cubic numbers.

18. If 30% of X is equal to 10% of Y, we can write that 0.3X = 0.1Y. From this equation, we find that Y = 3X. Thus Y is equal to 300% of X.

19. To transform K0 into K1, divide the line segment K0 into 3 segments of equal length, then draw an equilateral triangle that has the middle segment as its base. Remove the line segment that is the base of the triangle that was drawn and you will get K1. To transform K1 into K2, apply the same algorithm. As you can see, the length of K1 is 4/3 times longer than that of K0. That of K2 will be 4/3 times longer than that of K1. Each iteration of the algorithm produces a fractal line that has a length which is 4/3 times longer than the previous one. If K0 is 1 unit long, the length of K4 will be equal to 1 x $(4/3)^4$ which is 256/81. If line segment K0 is replaced by an equilateral triangle and the same algorithm is applied to the three sides of the triangle, we then get the Von Koch snowflake. This fractal curve and Peano's space-filling curve (first curve that completely fills a 2 dimensional space) are the first examples of fractal curves in history.



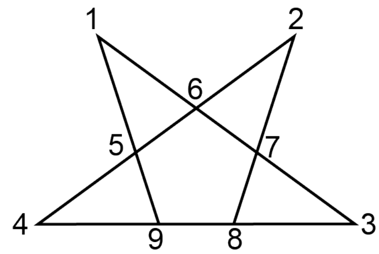
20. Since A x B = 21, B x C = 35, and C x D = 60, we can write that A x B x B x C x C x D = 21 x 35 x 60, which gives A x D x B x C x B x C = 44 100. This equation becomes A x D x 35 x 35 = 44 100 and finally A X D = 36.



21. The area of this solid is equal to twice the area of surface A, plus twice the area of surface B, plus the area of surface C, plus the area of the 2 faces of the cube that have not been touched when the triangular prism was cut away. The area of this solid is (2 x 34 + 2 x 24 + 6 x 2.83 + 2 x 36) closest to 205 cm^2.

22. In a tennis tournament, any player that loses a game is automatically eliminated. If there are 32 players participating, the champion plays exactly 5 games ($32 = 2^5$). If there are 20 players in the tournament, 10 games will be played in the first round to eliminate 10 players (one of the 10 winning players will eventually be the champion). The 10 winners will play 5 games in the 2nd round of play. In the 3rd round of play, there will be 5 players one of which will be the future champion. Since there is an odd number of players in the 3rd round, the best of these 5 players (as decided by the judges) will be able to skip the following rounds up until the best of the other 4 players is determined. During the 3rd round, 2 games will be played and 2 players will be eliminated. In the 4th round, 1 game will be played to determine the player that will go against the player that skipped the previous rounds. In the 5th round, the champion will be declared. The maximum number of games played by the champion will be 5, if and only if that champion is one of the players that played in all the rounds of play. We can rapidly compute the maximum number of games played by the champion (if the number of players is not an exact power of 2) by finding the power of 2, the largest possible, but whose value is less than the number of players in the tournament. The number of games played by the champion will be given by the exponent of that power of 2 to which you will add 1. In this tournament, there are 20 players and $20 = 2^4 + 4$. The maximum number of games played by the champion is (4 + 1) 5.

23. In the figure opposite, the vertices have been numbered from 1 to 9. Each triangle is represented by 3 numbers. There are 4 small triangles (like 1-5-6) and 3 larger triangles (like 2-4-8), each one composed of 2 small triangles and the central pentagon.

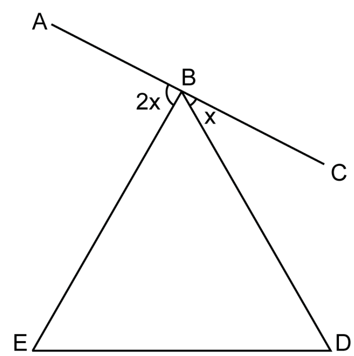


24. A letter is drawn at random from the name ‘’EULER’’. The probability of drawing a consonant is (2 favourable chances out of 5) 2/5.

25. The value of $2^{10} + 2^{10} + 2^{10} + 2^{10}$ is the same as 4×2^{10}, which is $2^2 \times 2^{10} = 2^{12}$.

$$\begin{array}{r} A\ B \\ \times\quad C \\ \hline F\ E\ D \end{array}$$

26. Each one of the 6 different letters in the diagram represents one of the following digits: 0, 1, 2, 3, 5, and 6. Evidently, letter C cannot be equal to 0 or 1. It cannot be equal to 3. If C = 3, then A cannot be equal to 0, 1, or 2 because whatever the value of B, the product of AB x 3 would be less than 100. Letter A cannot be 3 because letter C is equal to 3. Letter A cannot be 5. if A = 5, B cannot be 0 or 1 (in that case B and D would be equal), B cannot be equal to 2 (in that case A and E would be identical), B cannot take the values of 3 or 5 because C = 3 and A = 5. It cannot be 6 because in that case D would be equal to 8. We can prove in the same manner that letter A cannot be equal to 6 if C = 3. Letter C can be equal to 2 (53 x 2 = 106), 5 (32 x 5 = 160), and 6 (35 x 6 = 210).



27. The value of angle ABE is 2x. Since 2x + 60° + x is equal to 180°, we find that x = 40° and 2x = 80°.

28. The base of the triangle shown is tripled and its height is doubled. The area of the new triangle is thus (9 x 4 ÷ 2) 18. The area of the original triangle is (3 x 2 ÷ 2) 3. The area of the new triangle is (18 ÷ 3) 6 times greater than the area of the triangle opposite.