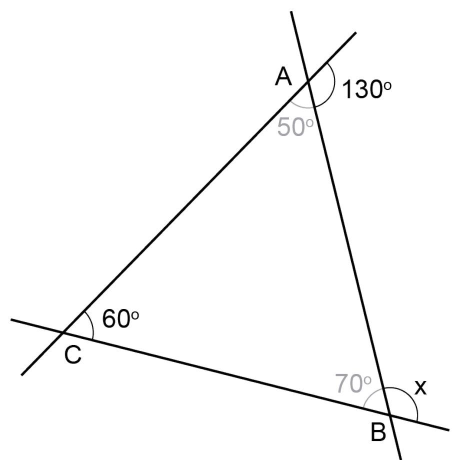
Mathematica Centrum

Together, let's shape the mathematicians of the future

NEWTON PREPARATORY TEST 2013
DETAILED SOLUTIONS

1. The cube root of the square of 8 (8^2 = 64) is equal to 4.

2. The value of (-2 + 6) - (-6 + 2) is 4 - (-4) = 8.



3. 1/3 + 1/2 + 1/6 = 2/6 + 3/6 + 1/6 = 6/6 = 1.

4. The value of x in the diagram opposite is (180º - 70º) 110º.

5. The smallest prime factor of 105 (3 x 5 x 7) is 3.

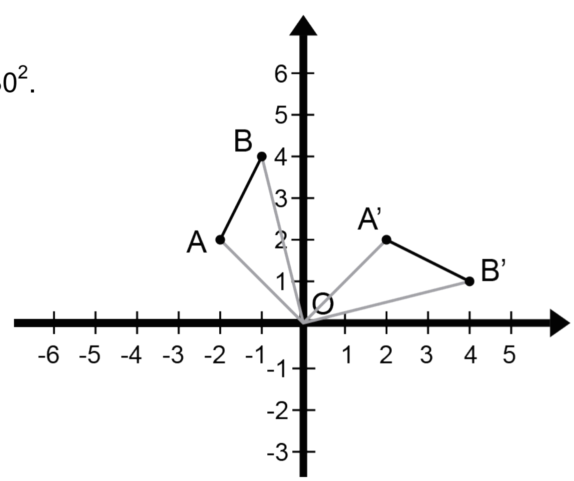
6. If n = √256 ÷ √81, then the value of n is 16/9 and that of √n is 4/3.

7. The result of 3/5 x 2/3 x 5/4 is 1/2 = 0.5.

8. 18% of 50 is equal to (0.18 x 50) 9. This value is equal to 9% of 100.

9. The number of minutes in 60 years (60 x 365 x 24 x 60) is the same as the number of seconds in one year (365 x 24 x 60 x 60).

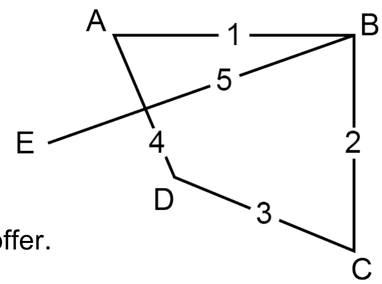
10. The result of $3^2 \times 5^2 + 3^2 \times 5^2 + 3^2 \times 5^2 + 3^2 \times 5^2$ is equal to $4 \times 3^2 \times 5^2 = 2^2 \times 3^2 \times 5^2 = 30^2$.



11. The coordinates of the images of points A and B of line segment AB if it is turned 90º clockwise around point O are A'(2, 2) and B'(4, 1).

12. If Mathilda's age is one third that of Mathusalem (who is 36 years older), we must conclude that 36 years = 2/3 of Mathusalem's age. Mathilda's age being one third that of Mathusalem, she must be 18 years old.

13. Points A, B, C, D, and E represent five North American cities. According to the diagram, the air route that goes from A to B is the same as the air route that goes from B to A. There are 4 air routes that leave or arrive at point A (AB, AC, AD, and AE). There are 3 more that leave or arrive at point B, 2 more from C and finally one last one from point D (those that leave or arrive at E are already included in the air routes that have been enumerated). In all, there are (4 + 3 + 2 + 1)10 different air routes that the company can offer.

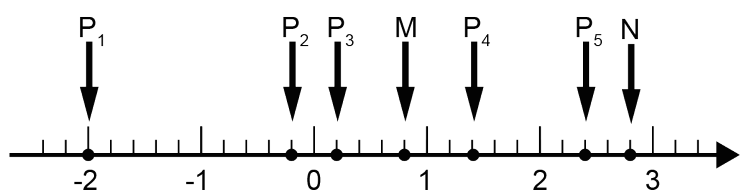


14. A 4-digit natural number is multiplied by a 2-digit natural number. The minimum number of digits that this product can have is (1 000 x 10) 5. The maximum number it can have is (9 999 x 99) 6. The product could have 6 digits.

15. The only number that is not prime is 9 (3 x 3).

16. The average of six numbers is 46. The total of these 6 numbers is (46 x 6) 276. If two of these numbers are 46 and 34, then the sum of the other 4 is (276 - 80) 196. The average of these 4 numbers is (196 ÷ 4) 49.

17. The point on the number line which is 4 times further from point M than from point N is point P_5. Indeed, P_5 is at a 2/10 of one unit distance from N and at a 8/10 of one unit distance from M.



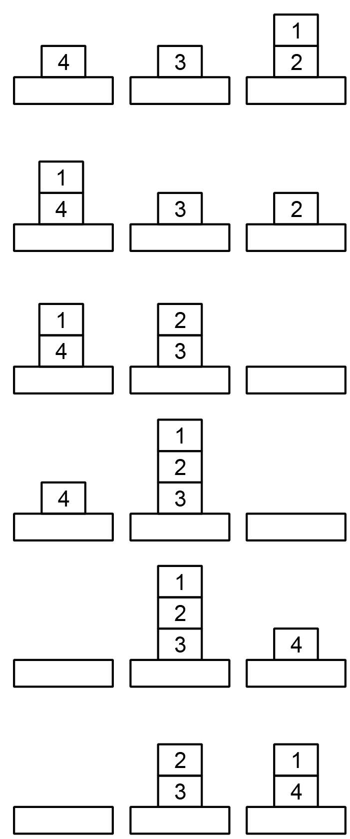
18. The number which is a multiple of 6, but is not a multiple of 5 is 186 (2 x 3 x 31).

19. Each 11 cm x 17 cm sheet of paper can give four 5 cm x 8 cm sheets of paper. One hundred 11 cm x 17 cm sheets of paper will give (4 x 100) 400 sheets 5 cm x 8 cm.

20. X is not second. He could be 1st, 3rd or 4th. He cannot be 4th because Y is just behind him. W could be 2nd or 3rd. If W were 3rd, Z would be 2nd (because he is just in front), X would be 1st and Y and Z would both be 2nd. W must be 2nd, Z must be 1st, X 3rd and Y last.

21. To make a box 9 cm high, Carol must cut away a small 9 cm square carton at each corner of the rectangular carton. In order to make this box, Carol had to cut away an area of (4 x 81 cm^2) 324 cm^2 from the large carton.

22. He must make a minimum of 5 moves in order to stack the blocks as shown in the last figure of the diagram opposite.



23. The number 12! contains the factor 3 (3 = 1 x $\underline{3}$, 6 = 2 x $\underline{3}$, 9 = $\underline{3}$ x $\underline{3}$, and 12 = $\underline{3}$ x 4) 5 times. The maximum value of k for which 3^k is a factor of 12! is 5.

24. When Melissa hangs her hat, she has a choice of 4 hooks. Andrea will only have a choice of 3 hooks when she hangs hers. They can hang their hats in (3 x 4) 12 different ways.

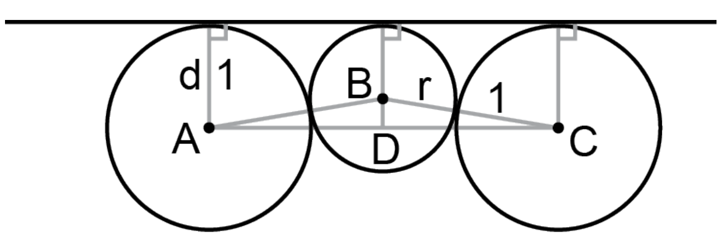
25. These palindrome numbers all have 3 digits. There are 9 different digits that can be chosen for the hundreds' place (these numbers cannot begin with the digit 0); there are 10 different digits that can be chosen for the tens' place. We have no choice of digits for the ones' place, because these digits must be the same as those in the hundreds' place. In all, there are (9 x 10) 90 different palindrome numbers between 100 and 1 000.

26. In this card trick, the sum of points is a multiple of 10. Indeed, 1 + 9 = 10, 2 + 8 = 10, 3 + 7 = 10, 4 + 6 = 10, and as the same value turns up 4 times, the total sum of all the cards with a face value of 1 to 9 is a multiple of 10 (the sum of the four ''5's'' is also a multiple of 10). As all the other cards are worth 10 points each, the total number of points for all 52 cards is also a multiple of 10. This sum is 340. As the sum of the first 44 cards is a multiple of 10, the sum of the last 8 cards must also be a multiple of 10. Given that the sum of the last 7 cards is (3 + 6 + 1 + 10 + 10 + 4 + 8) 42, the value of the 52^{nd} card dealt was (50 - 42) 8. In reality, to guess the value of the last card dealt, Mathusalem uses modulo 10 arithmetic. How? Let's suppose that first card dealt is a 7 and the second one is a 4. Instead of keeping in mind the sum of 11, he keeps in mind the value of 1 because he knows that $11 \equiv 1 \bmod 10$. Every time that the sum is greater than 10, he only keeps in mind the remainder of the division by 10 because he knows that the value of the last will be given by ''10 minus the value that he will have in mind just before receiving the last card''. Of course, if the first card dealt is a 7 and the second is a 2, he will keep in mind the value of 9 because he has not reached the modulus which is 10. Don't forget that Mathusalem uses modulo 10 arithmetic only when the value in his mind added to the value of the next card dealt yields a sum that is greater than 10 (by the way, if the sum is equal to 10, he keeps in mind the value of 0 because $0 \equiv 10 \bmod 10$).

27. The maximum number of operas that he could have seen is 7, if he saw an opera on the first day of the 49 day period.

28. The power 2/3 in $8^{2/3}$ has the following meaning: the 2 (the numerator) indicates that we must square the 8 and the 3 (the denominator) that we must find the cube root of the result. The value of $8^{2/3}$ is 4.

29. When the straight line is tangent to the 3 circles, we can write that $(1 + r)^2 = (1 - r)^2 + (3/2)^2$ ($BC^2 = BD^2 + DC^2$). This equation becomes $r^2 + 2r + 1 = r^2 - 2r + 1 + 9/4$ and we find that r = 9/16. If d is the distance between line segment AC and the tangent, we can say that the line is tangent to the 3 circles when the value of d is 1.



30. To find the perimeter of triangle ABC, we must first find the coordinates of the 3 vertices of the triangle, then the lengths of its 3 sides. We find the coordinates of A by solving the system of equations $y = 2$ and $y = x + 7$. We find A(-5, 2). We find the coordinates of B by solving the system of equations $y = -2x + 4$ and $y = x + 7$. We find B(-1, 6). We find the coordinates of point C by solving the system of equations $y = -2x + 4$ and $y = 2$. We find C(1, 2). The length of line segment AC is (1 -(-5)) 6. The length of AB is given by $AB^2 = (6 - 2)^2 + (-1 - (-5))^2$. We find that $AB = 4\sqrt{2}$. Equation $BC^2 = (2 - 6)^2 + (1 - (-1))^2$ yields the value of BC which is $2\sqrt{5}$. The value closest to the perimeter of triangle ABC is $(6 + 4\sqrt{2} + 2\sqrt{5})$ 16.1.