

# Mathematica Centrum

Together, let's shape the mathematicians of the future

## LAGRANGE PREPARATORY 2014

- The value of  $n$  in the equation:  $n \times 5\% = 100$  is  
A) 500                      B) 10                      C) 1 000                      D) 100                      E) 2 000
- $\frac{3}{4}$  of  $\frac{1}{4}$  of 16 = ?  
A) 3                      B) 6                      C) 4                      D) 2                      E) 5
- The value of  $(-2 - 5) + (-5 - 3)$  is  
A) 10                      B) 0                      C) -15                      D) -9                      E) -18
- $2 \times 3 - 5 \times -3 = ?$   
A) 21                      B) -9                      C) -21                      D) 12                      E) -3
- $(\frac{1}{6} - \frac{1}{3}) \times \frac{1}{5} = ?$   
A)  $-\frac{1}{36}$                       B)  $\frac{7}{30}$                       C)  $\frac{1}{16}$                       D)  $-\frac{1}{30}$                       E)  $\frac{1}{60}$
- The product of 4 prime numbers is never smaller than  
A) 360                      B) 210                      C) 220                      D) 305                      E) 260
- The sum of all natural numbers smaller than 49 that are square numbers is  
A) 126                      B) 90                      C) 91                      D) 88                      E) 89
- Which of the following is closest to a speed of 60 km/h?  
A) 16 m/s                      B) 17 m/s                      C) 15 m/s                      D) 18 m/s                      E) 19 m/s
- Mathusalem has chosen 4 different integers between 1 and 20. If their product is 110, what is their sum?  
A) 19                      B) 20                      C) 18                      D) 17                      E) impossible

10. It takes 6 minutes to fill  $\frac{3}{7}$  of a bath-tub. At this rate, the number of extra minutes needed to fill another seventh ( $\frac{1}{7}$ ) is

- A) 2.5 minutes    B) 4 minutes    C) 8 minutes    D) 2 minutes    E) 3 minutes

11. One quarter of 3 hours and 20 minutes is equal to

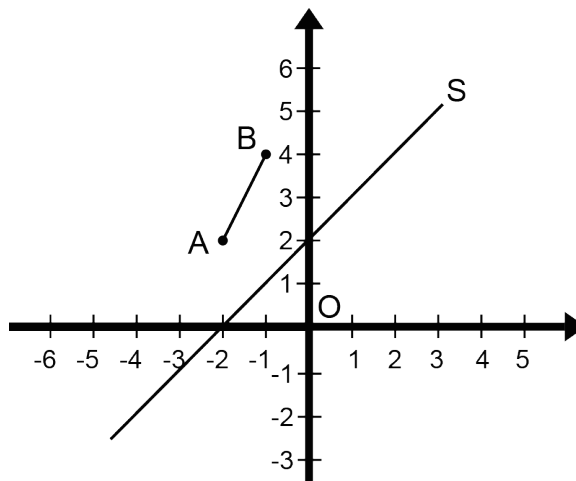
- A) 35min    B) 40min  
C) 45min    D) 55min  
E) 50min

12. Which of the following has the least factors?

- A) 12    B) 21    C) 16  
D) 20    E) 24

13. What are the coordinates of the image of point A of line segment AB if it is reflected in the S-axis?

- A)  $A'(0, 0)$     B)  $A'(1, -1)$   
C)  $A'(1, 1)$     D)  $A'(2, 2)$     E)  $A'(-1, 1)$

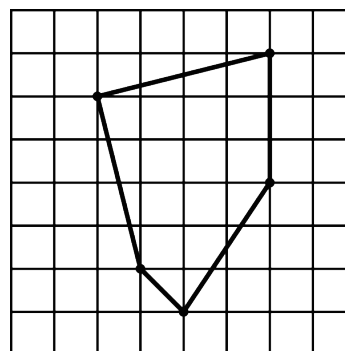


14. What is the smallest positive integer that multiplied by 12 will yield a square number?

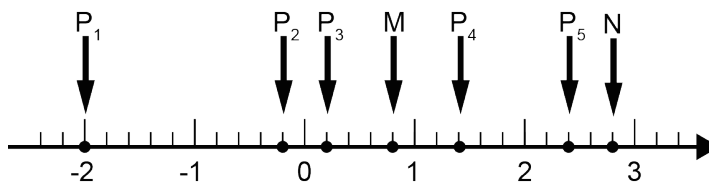
- A) 6    B) 5    C) 4  
D) 3    E) 2

15. What is the area of the pentagon in the diagram opposite, if the area of each small square is equal to  $1 \text{ cm}^2$  and if each vertex of the pentagon coincides with a vertex of a square?

- A)  $16.5 \text{ cm}^2$     B)  $17 \text{ cm}^2$     C)  $15.5 \text{ cm}^2$   
D)  $17.5 \text{ cm}^2$     E)  $16 \text{ cm}^2$



16. Which point on the number line represents the average of points  $P_1$  and  $P_5$ ?



- A) M    B) N    C)  $P_3$     D)  $P_2$     E)  $P_4$

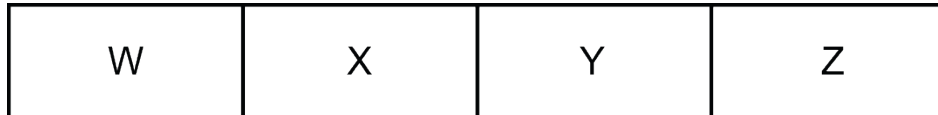
17. A happy number is defined by the following process: Starting with any natural number greater than 0, replace the number by the sum of the squares of its digits. Repeat the process until the number obtained equals 1. If the process loops endlessly in a cycle (endless cycle) that generates the same sequence of numbers which does not include 1, we say that the number is an unhappy number. Which of the following is a happy number?

- A) 4    B) 3    C) 6    D) 7    E) 5

18. The area of a rectangle is equal to the area of a square. The side of the square and the base of the rectangle are doubled. If  $c$  represents the side of the initial square, which of the following represents the difference between the area of the new square and that of the new rectangle?

- A) 0                      B)  $2c^2$                       C)  $c^2$                       D)  $3c^2$                       E)  $4c^2$

19. Last week, four people (W, X, Y, and Z) were sitting on a bench as shown in the diagram below. Today, they are sitting in a different way. W is not sitting beside X nor Y. Y is sitting beside Z. Today, which people are sitting between the other two?

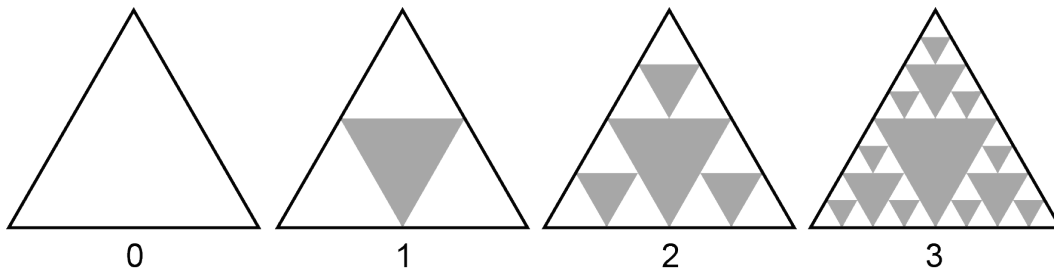


- A) X and Z                      B) Y and W                      C) Z and Y                      D) X and Y                      E) W and X

20. If  $x = -3$ , what is the value of  $x + x^2 + x^3$ ?

- A) -21                      B) 33                      C) -3                      D) 21                      E) -39

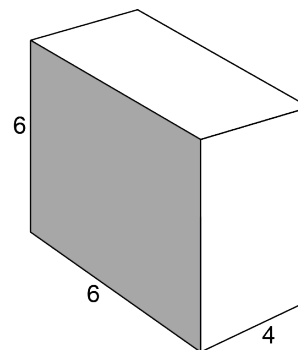
21. Look carefully at the diagram below which represents the first 4 steps (step 0, step 1, step 2, and step 3) of the transformation of a triangle. How many triangles will be removed on the fifth step (step 4) of the transformation (Sierpinski's fractal)?



- A) 21                      B) 24  
C) 18                      D) 27  
E) 15

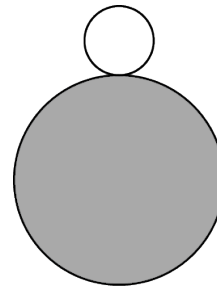
22. All the measures in the diagram opposite are in centimetres. What is the ratio (in  $\text{cm}^2$  per  $\text{cm}^3$ ) of the total area of this rectangular prism compared to its volume?

- A)  $8/7$                       B)  $6/7$   
C)  $7/8$                       D) 1  
E)  $7/6$

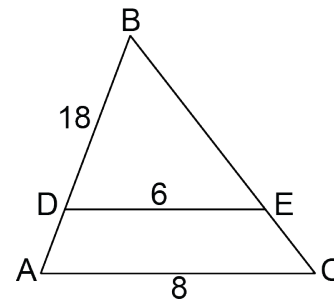


23. If  $M_1 = 12$ ,  $N_1 = 8$ ,  $M_2 = 16$ , and  $N_2 = 24$  and if  $P_1$  and  $G_1$  are the LCM and the GCD of  $M_1$  and  $N_1$  and if  $P_2$  and  $G_2$  are the LCM and the GCD of  $M_2$  and  $N_2$ , then the product  $P_1 \times P_2 \times G_1 \times G_2$  is equal to
- A) 36 864      B) 18 432      C) 12 288      D) 24 576      E) 55 296
24. If  $n$  is a positive integer and  $n^2 + 3$  is odd, which of the following is always even?
- A)  $n + 3$       B)  $n^2 + 1$       C)  $n^3 - n$       D)  $n + 1$       E)  $3n - 1$
25. The sum of  $n$  positive integers is equal to 8. If  $P$  represents the product of these  $n$  numbers, what is the greatest possible value of  $P$ ?
- A) 15      B) 10      C) 12      D) 16      E) 18
26. The number of even factors of 12 is equal to
- A) 3      B) 4      C) 5      D) 6      E) 2
27. If  $A \times B = 12$ ,  $B \times C = 20$ , and  $C \times D = 40$ , what is the value of  $A \times D$ ?
- A) 24      B) 18      C) 20      D) 30      E) 32
28. The measures of three of the four angles of a quadrilateral are in the ratio 2 : 3 : 7. The sum of these 3 angles is equal to  $240^\circ$ . What is the value of the largest angle of this quadrilateral?
- A)  $140^\circ$       B)  $110^\circ$       C)  $120^\circ$   
D)  $130^\circ$       E)  $100^\circ$

29. A circular coin, with a radius of 1, turns without slipping around a circular coin with a radius of 3. How many revolutions will the small coin have completed when it comes back to its initial position?
- A) 2      B) 3      C) 4  
D) 5      E) 6



30. In triangle ABC opposite, line segment DE is parallel to side AC. If all measures are in centimetres, what is the length of side AB?
- A) 20 cm      B) 18 cm  
C) 21 cm      D) 16 cm  
E) 24 cm



31. The rule of a sequence is  $3n - 2$ . If  $n$  is a positive integer, how many terms of this sequence are even and less than 100?
- A) 32      B) 15      C) 17      D) 16      E) 33
32. If  $2\pi R = \pi R^2$  with  $R \neq 0$ , then  $R = ?$
- A) 1      B) 2      C) 3      D) 4      E) 5