Mathematica Centrum

Together, let's shape the mathematicians of the future

NEWTON PREPARATORY TEST 2014
DETAILED SOLUTIONS

1. The value of n in the equation: n x 5% = 100 is (100 ÷ 0,05) 2 000.

2. 3/4 of 1/4 of 16 = 3/16 x 16 = 3.

3. The value of (-2 - 5) + (-5 - 3) is (-7 + -8 = -7 -8) -15.

4. 2 x 3 - 5 x -3 = 6 + 15 = 21.

5. (1/6 - 1/3) x 1/5 = (1/6 - 2/6) x 1/5 = -1/6 x 1/5 = -1/30.

6. The product of 4 prime numbers is never smaller than (2 x 3 x 5 x 7) 210.

7. The sum of all natural numbers less than 49 that are square numbers is (1 + 4 + 9 + 16 + 25 + 36) 91.

8. A speed of 60 km/h is closest to (60 x 1 000 m ÷ 3 600 s = 16,666 …) 17m/s.

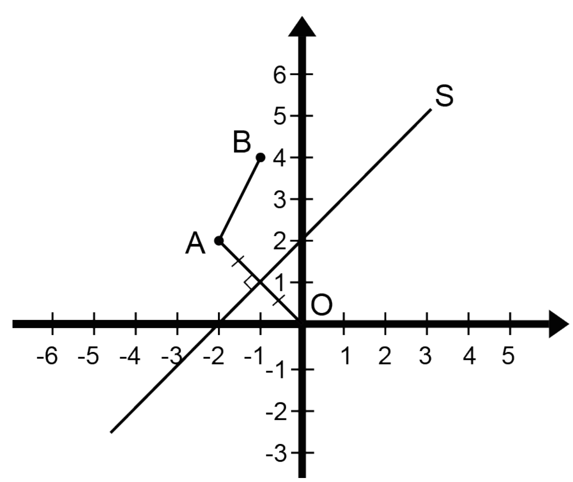
9. The product 110 can only be written has 2 x 5 x 11. Mathusalem could not have chosen 4 different integers between 1 and 20. The problem is impossible.

10. It takes 6 minutes to fill 3/7 of a bath-tub. At this rate, the number of extra minutes needed to fill another seventh (1/7) is (6 ÷ 3) 2 minutes.

11. One quarter of 3 hours and 20 minutes is equal to ((3 x 60 + 20) ÷ 4) 50 minutes.

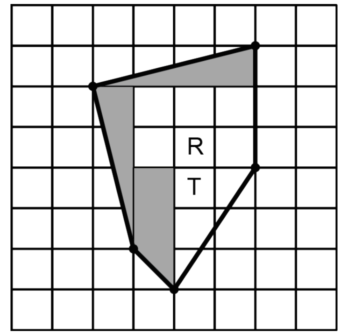
12. Number 21 has only 4 factors (1, 3, 7, and 21). The number 16 has five (1, 2, 4, 8, and 16). All the other numbers have more than 5 factors.

13. The image of point A of line segment AB, if S is a flip line, is at O(0, 0). Line segment AO is perpendicular to S (see diagram opposite).

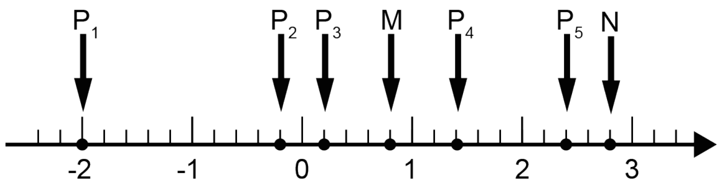


14. The smallest positive integer that, when multiplied by 12 will yield a square number is (12 x 3 = 36) 3.

15. The area of the pentagon opposite is equal to the sum of the areas of the two shaded triangles, of rectangle R, of the shaded trapezium, and of triangle T. This sum is equal to (2 cm^2 + 2cm^2 + 6 cm^2 + 2.5 cm^2 + 3 cm^2) 15.5 cm^2.



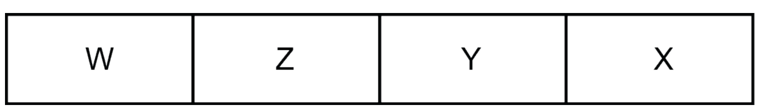
15. The point representing the average of P_1 and P_5 is P_3 because it is equal to ((-2 + 2.4) ÷ 2) 0.2.



17. Number 7 is a happy number because 7^2 = 49, $4^2 + 9^2$ = 97, $9^2 + 7^2$ = 130, $1^2 + 3^2 + 0^2$ = 10, and $1^2 + 0^2$ = 1.

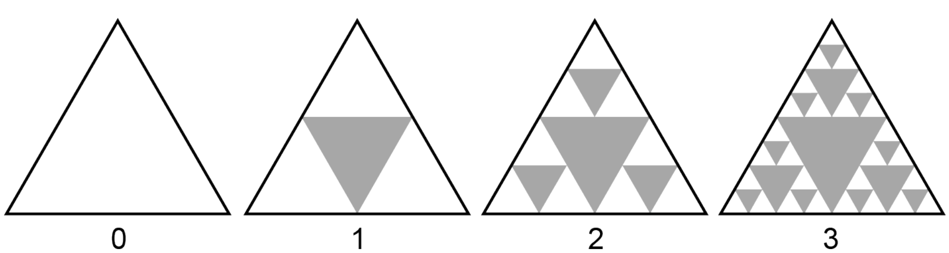
18. The area of the rectangle is equal to the area of a square. The side of the square and the base of the rectangle are doubled. If c represents the side of the initial square, the area of the new square is ($(2c)^2$) $4c^2$, that of the new rectangle is (2 x c^2) $2c^2$. The difference between the area of the new square and that of the new rectangle is ($4c^2 - 2c^2$) $2c^2$.

19. Since W is not sitting beside X nor Y, it must be sitting at one of the two ends. Let's place him on the left end of the bench. Z must be sitting just to the right of W because X and Y cannot be sitting beside W. Y must be sitting just to the right of Z and X is sitting on the right end of the bench. Today, Z and Y are sitting between the other two.

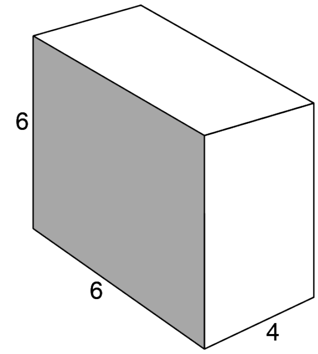


20. If x = -3, the value of $x + x^2 + x^3$ is equal to $(-3 + (-3)^2 + (-3)^3)$ -21.

21. On step 1, (3^0) one triangle is removed, on step 2 (3^1) 3 triangles are removed, on step 3 (3^2) 9 triangles are removed. The number of triangles that will be removed on the fifth step (step 4) is equal to (3^3) 27.



22. The total area of this prism is ($2 \times 36\ cm^2 + 2 \times 24\ cm^2 + 2 \times 24\ cm^2$) $168\ cm^2$. Its volume is ($36\ cm^2 \times 4\ cm$) $144\ cm^3$. The ratio (in cm^2 by cm^3) of the total area compared to the volume is ($168\ cm^2 \div 144 cm^3$) $7/6\ cm^2/cm^3$.



23. If $M_1 = 12$, $N_1 = 8$, $M_2 = 16$, and $N_2 = 24$, then $P_1 = 24$, $G_1 = 4$, $P_2 = 48$, and $G_2 = 8$. The product $P_1 \times P_2 \times G_1 \times G_2$ is equal to ($24 \times 48 \times 4 \times 8$) 36 864.

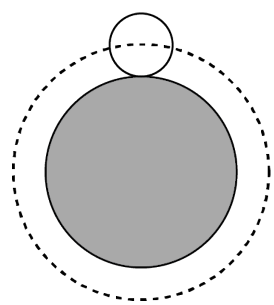
24. If n is a positive integer and $n^2 + 3$ is odd, then n^2 is even and consequently n is even. The expression $n^3 - n$ is always even.

25. The sum of n positive integers is equal to 8. If P represents the product of these n numbers, the greatest possible value of P is ($3 \times 3 \times 2$) 18.

26. The factors of 12 are 1, 2, 3, 4, 6, and 12. The number of even factors of 12 is equal to 4.

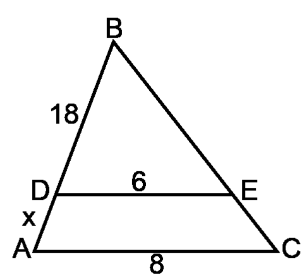
27. If $A \times B = 12$, $B \times C = 20$, and $C \times D = 40$, we can write that $\underline{A \times B} \times \underline{B \times C} \times \underline{C \times D} = 9\,600$. This equation can be written as $A \times D \times (BC^2) = 9\,600$, from which, we find the value of $A \times D$ which is equal to 24.

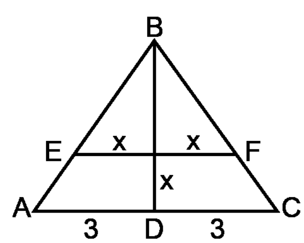
28. The measures of three of the four angles of a quadrilateral are in the ratio 2 : 3 : 7. Since the sum of these 3 angles is equal to 240º, we can write that $2x + 3x + 7x = 240°$. We find that $x = 20°$ and that the three angles are 40°, 60° and 140°. Since the sum of the 4 angles of a quadrilateral is 360° (the sum of the angles of two triangles), we find that the value of the 4th angle is (360° - 240°) 120°. The value of the largest angle of this quadrilateral is 140°.



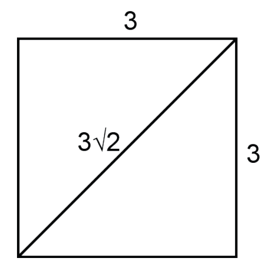
29. A circular coin, with a radius of 1, turns (without slipping) around a circular coin with a radius of 3. The distance covered by the small coin is equal to the distance covered by its centre. This centre moves a distance which is equal to ($2 \times \pi \times 4$) 8π. The number of revolutions that the small coin will have completed when it comes back to its initial position is ($8\pi \div 2\pi$) 4.

30. In triangle ABC opposite, line segment DE is parallel to side AC. Since DE is parallel to AC, triangle BED is similar to triangle ABC (point B is the centre of homothety). We can write that $(x + 18) : 18 = 8 : 6$. From this equation, we find that $x = 6$ and that the length of AB is equal to (6 + 18) 24 cm. In reality, all corresponding sides of these 2 similar triangles are in a ratio of 8 : 6. We can compare the heights, the medians or any two corresponding lines. For example, in the second figure opposite where the two triangles are similar (EF is parallel to AC) and where the height BD of the large triangle is 4, we can find the value of x by comparing the heights and the bases of triangles ABC and EBF. We can write that $(4 : (4 - x)) = 6 : 2x$. We find that $x = 12/7$.



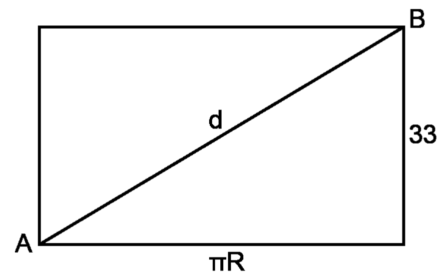


31. The average distance between two vertices of a square of side 3 is equal to $((3 + 3 + 3\sqrt{2}) \div 3)$ $2 + \sqrt{2}$.

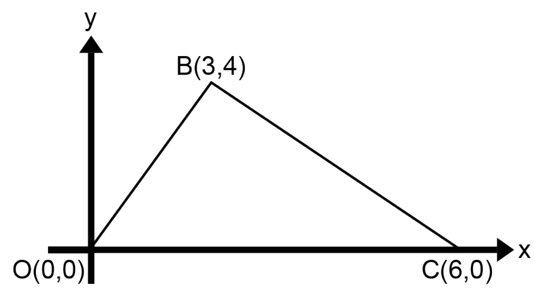


32. If $y = 2x$ and $x + 2y = 30$, and by substituting the first equation into the second, we can write that $x + 4x = 30$ and find that $x = 6$ and $y = 12$. The value of $x^2 + y^2$ is equal to $(6^2 + 12^2)$ 180.

33. The shortest path is given by the straight line between point A and point B, when the cylindrical surface is developed into a plane surface. Line segment AB becomes the hypotenuse of the right angle triangle whose legs are the half circumference ($\pi R = 43.98$ cm) and the vertical distance between the two points (33 cm). The length of this hypotenuse is almost $(\sqrt{(33^2 + 44^2)})$ 55 cm. We can find this distance more easily if one remembers that the triangle formed, when the cylindrical surface is developed is, in reality, the most famous of all right angle triangles, triangle 3 : 4 : 5 (which in this problem is a little bit disguised). Indeed, the right angle triangle of this problem can be written as: $(\mathbf{3} \times 11)^2 + (\mathbf{4} \times 11)^2 = (\mathbf{5} \times 11)^2$. For your information, all the lengths of helicoidal stairs that wrap around the towers that we see when we pass close to a chemical plant are calculated in this way.



34. The product of the slopes (rate of change) of sides OB and BC of triangle OBC opposite is equal to $(4/3 \times -4/3)$ $-16/9$.



35. The value of $5! + 2$ is $(120 + 2)$ 122.