Mathematica Centrum

Together, let's shape the mathematicians of the future

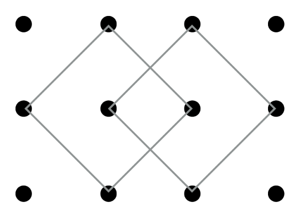
NEWTON PREPARATORY TEST 2015
COMPLETE SOLUTIONS

1. The value of N in the equation: -4 x -2 + N = -4 is (8 + N = -4 and N = -12) -12.

2. The natural numbers between 1 and 15 that have only 2 factors are: 2, 3, 5, 7, 11, and 13. These numbers are all the prime numbers between 1 and 15.

3. The number 2^9 (512) is closest to 500.

4. The number of squares (of all sizes) whose vertices coincide with the points in the diagram is equal to 10. There are 6 small 1 x 1 squares, two 2 x 2 squares, and two other squares that are shown in the diagram.



5. 25% of a number is equal to (0.8 x 200) 160. This number is equal to (4 x 160) 640 and twice the same number is equal to (2 x 640) 1 280.

6. The sum of all the prime factors of 50 {2, 5, 5} is equal to 12.

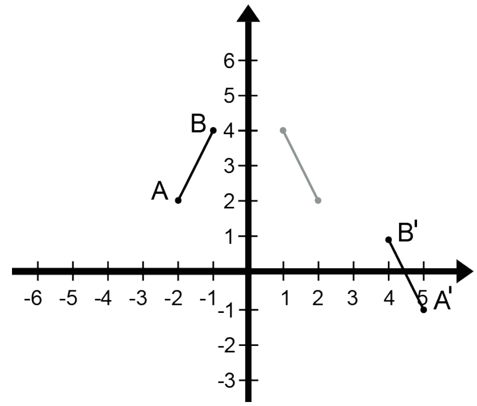
7. The minimum number of biscuits that she can sell is the LCM of 10 and 8. The LCM of 10 and 8 is (2 x 2 x 2 x 5) 40.

8. Mathew has paid $6.30 for **2** hot dogs and **3** fries. Mathilda has paid $6.20 for **3** hot dogs and **2** fries. For **5** hot dogs and **5** fries, they should pay ($6.30 + $6.20) $12.50.

9. The perimeter of a rectangle is equal to 28 cm. The largest possible area of a rectangle having a certain perimeter always happens when its length is equal to its width. The largest possible area is equal to the area of the square whose side measures (28 ÷ 4) 7 cm. This area is equal to (7 cm x 7 cm) 49 cm^2.

10. The central number (the one in the middle) of 5 consecutive natural numbers always represents the average of the 5 numbers. If this number is N, we can say that the sum of these 5 numbers is 5N. The sum of these 5 consecutive natural numbers is always a multiple of 5 (5N). The largest natural number, less than 50, that is equal to the sum of 5 consecutive natural numbers is thus 45.

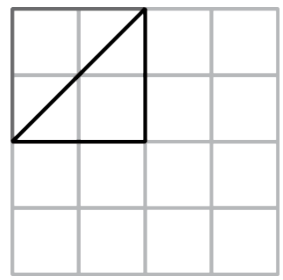
11. The coordinates of the image of point A of line segment AB after it is reflected in the y-axis and is moved (translation) by a value of t (3, -3) is (5, -1).



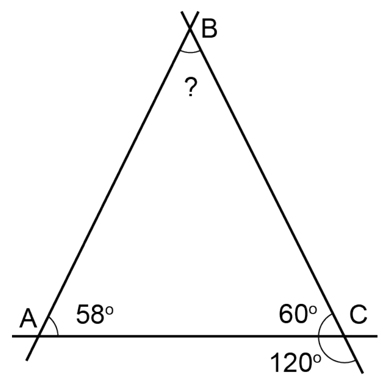
12. Starting with 0, all integers are written in increasing order: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, … . From 0 to 9, 10 digits are written; from 10 to 19, 20 digits are written; from 20 to 29, 20 more digits are written. The 50^{th} digit that will be written is a 9.

13. The value of (80%)% is (0.8)% or 0.008.

14. First a triangle, like the one shown in the diagram, can be traced inside the 2 x 2 bold square. By symmetry or rotation, we can trace 3 other triangles in the same bold square. By downward translations of these 4 triangles, we can trace 8 more triangles. By translations to the right of these 12 triangles, we can trace 24 more triangles. The maximum number of isosceles right-triangles having the same dimensions that could be drawn in this grid is 36.



15. The measure of angle ABC is equal to (180º - (60º + 58º)) 62º.



16. The diagram on the right hereafter represents the universe of all the possible sums (36) when two dice are rolled. The bold pairs (12) represent the cases where the sum of the two dice is smaller than 7. The probability of getting a sum smaller than 7 is (15/36) 5/12.

(1,1)	**(1,2)**	**(1,3)**	**(1,4)**	**(1,5)**	(1,6)
(2,1)	**(2,2)**	**(2,3)**	**(2,4)**	(2,5)	(2,6)
(3,1)	**(3,2)**	**(3,3)**	(3,4)	(3,5)	(3,6)
(4,1)	**(4,2)**	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

17. The average of two integers is -2. The sum of these two integers is (2 x -2) -4. The sum of these two integers and the number -5 is equal to (-4 + -5) -9.

18. A team has 20 players. Five have blond hair, 12 have blue eyes, and 4 of them have both blond hair and blue eyes. Because 4 of these players have both blond hair and blue eyes, we can conclude that (5 - 4) 1 of these players has blond hair but not blue eyes and (12 - 4) 8 of these players have blue eyes but not blond hair. The number of players that have neither blue eyes nor blond hair is equal to (20 - (1 + 8 + 4)) 7.

19. The value of the numerator so that the fraction is equal to 3 is 3n + 3.

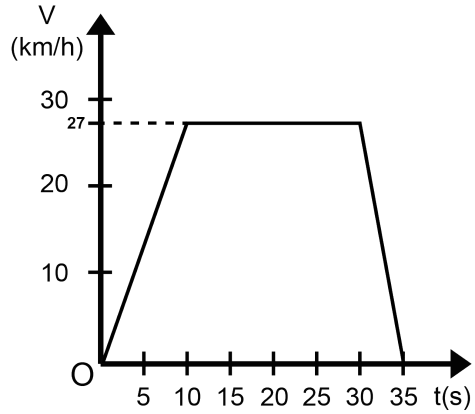
20. The number of rectangular cereal boxes of 7 cm x 25 cm x 50 cm that can be stacked in a rectangular box of 49 cm x 100 cm x 151 cm is (7 x 4 x 3) 84.

21. If x = -2, the value of $2x - x^2 - x^3$ is $(2(-2) - (-2)^2 - (-2)^3 = -4 - 4 + 8)$ 0.

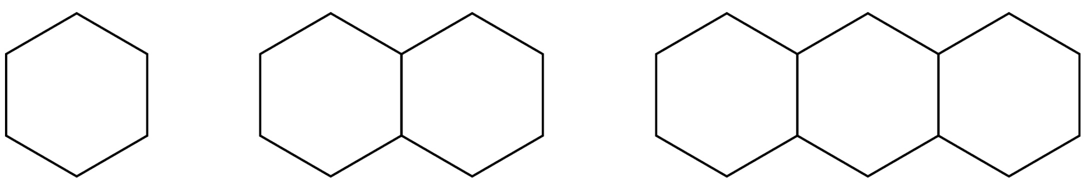
22. The sum of the numbers that form its 7th line is (2 x 1 + 2 x 6 + 2 x 15 + 20) 64.

1						
1	1					
1	2	1				
1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	
1	6	15	20	15	6	1

23. The graph shows the relation between the speed V (in km/h) and the time t (in seconds) of a car that has moved for 35 seconds. From rest, the car increased its speed up to 27 km/h, a speed which it kept for a certain time. It then started slowing down and finally came to a stop. This car kept a constant speed for a period of time (30 - 10) of 20 seconds.



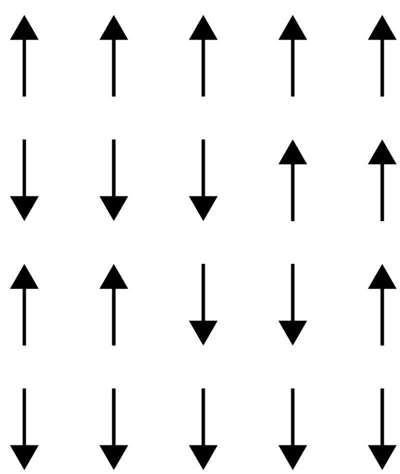
24. A sequence of figures composed of hexagons is shown below. The perimeter of each figure (of each term in the sequence) is indicated under the figure. The perimeter of the first figure is 6, that of the 2nd figure is (6 + 1 x 4) 10, that of the 3rd figure is (6 + 2 x 4) 14. The perimeter of the figure of rank n is 6 + 4(n - 1). The perimeter of the 20th figure (20th term) in this sequence is thus 6 + 4(20 - 1) 82.



6 10 14

25. If n is a positive integer and 5 < n < 12, there are 5 different values of n for which there is a triangle with sides of lengths 3, 8, and n. These triangles are 3 - 8 - 6, 3 - 8 - 7, 3 - 8 - 8, 3 - 8 - 9, and 3 - 8 - 10.

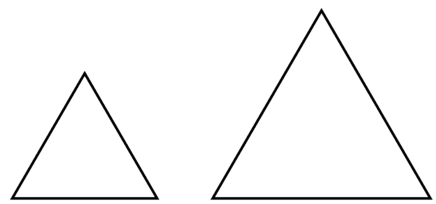
26. Mathusalem just washed 5 wine glasses that are right side up and wants to turn them over so that they can dry faster. By turning 3 glasses at a time, he wants to get 5 glasses that are upside down. A glass that is upside down can be turned right side up. If the inversion of 3 glasses is equivalent to one operation, we can complete the task in 3 operations, as shown in the diagram.



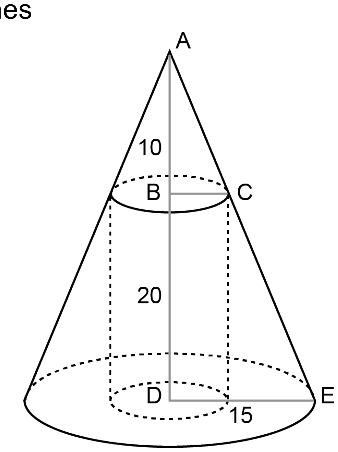
27. Let x be the age of Mathilda. Mathew's age is presently x + 15 years. In 10 years, Mathew's age will be x + 25 years and Mathilda's age will be x + 10 years. We know that x + 25 = 2(x + 10). From this equation, we find that the respective ages of Mathilda and Mathew are presently 5 years old and 15 years old. In 20 years, Mathilda will be (5 + 20) 25 years old.

	Mathew	Mathilda
Present	x + 15	x
Future	x + 25	x + 10

28. An astronaut is moving around a perfectly spherical planet that has a radius of R km. If the astronaut measures exactly 2 metres, her head would move a distance of 30º/360º(2π(R + 2)). Her feet would move a distance of 30º/360º(2πR). Her head would move 30º/360º((2π(R + 2) - 2πR)) more than her feet. This distance represents (1/12(2πR + 4π - 2πR)) 1/12 x 4π or π/3 m.



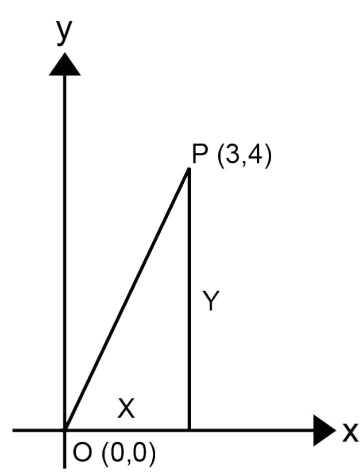
29. The perimeter of the large equilateral triangle is 2 times larger than the perimeter of the small equilateral triangle. Knowing that 2 equilateral triangles are always similar, we can say that the area of the large triangle is K^2 times larger than the small one. The area of the large triangle is ($K^2 = 2^2$) 4 times larger than the area of the small triangle?



30. Triangle ABC is similar to triangle ADE. This means that AB : AD = BC : DE or BC = 10 x 15 ÷ 30 = 5. The radius of the cylinder (BC) is 5 cm.

31. The volume of the cube is (3 cm x 3 cm x 3 cm) 27 cm^3. The volume of the hole is (1 cm x 1 cm x 3 cm) 3 cm^3. The volume of the solid with the hole is (27 - 3) 24 cm^3.

32. Using the decimal system to write all the zeros in the number 10^{google}, we would have to write a google zeros. At a rate of 1 zero per second it would take a google seconds. Since there are about (365 x 24 x 60 x 60) 31 536 000 seconds in a year, it would take you approximately (10^{100} ÷ 3.15 x 10^7 or 3.17 x 10^{92}) 10^{92} years.



33. The coordinates of point P are (3, 4). If point O is the origin (0, 0) of the Cartesian plane, the length of line segment OP can be found by using Pythagoras' theorem. From $X^2 + Y^2 = OP^2$, we can write that $OP = \sqrt{(3^2 + 4^2)}$. We find OP = 5.

34. If $x^2 + y^2 = 5$ and if y = 2x, we can say that $x^2 + (2x)^2 = 5$ and that $x^2 + 4x^2 = 5$ or $5x^2 = 5$. We find that x = 1 and y = 2. The value of x + y is equal to (1 + 2) 3.