Mathematica Centrum

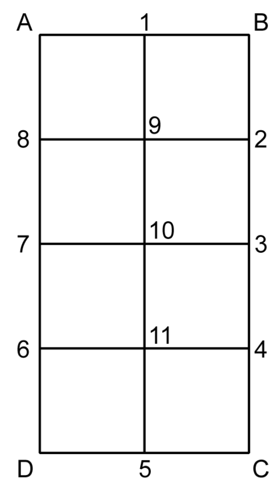
Together, let's shape the mathematicians of the future

EULER PREPARATORY TEST 2016
DETAILED SOLUTIONS

1. The prime factors of 333 are {3, 3, 37}. The largest prime factor of 333 is 37.

2. Two of the numbers, 1 and 64, are perfect squares and cubes. Indeed, $64 = 8^2 = 4^3$ and $1 = 1^2 = 1^3$.

3. So as not to forget any rectangle, we have numbered the vertices of the possible rectangles. There are 7 rectangles whose bases are 2 units long. These are A-B-2-8, A-B-4-6, A-B-C-D (the original rectangle itself), 8-2-3-7, 8-2-C-D, 7-3-4-6, and 6-4-C-D. There are 12 rectangles which have a base that is 1 unit long. These are A-1-10-7, A-1-11-6, A-1-5-D, 8-9-11-6, 8-9-5-D, 7-10-5-D and their 6 symmetrical rectangles 1-B-3-10, 1-B-4-11, 1-B-C-5, 9-2-4-11, 9-2-C-5, and 10-3-C-5. In all, we can count 19 rectangles.



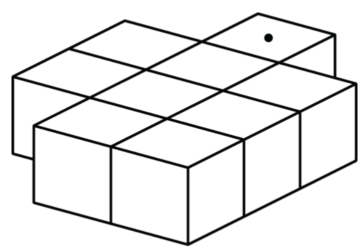
4. Twenty-seven (3 x 3 x 3) cubes with edges 2 cm long are needed to form a cube with edges 6 cm long.

5. The number Z, representing the average of the other four choices, must satisfy the conditions of equation: Z x 4 = sum of the other 4. This number is -3, because -3 x 4 = 4 + (-4) + (-17) + 5.

6. I gave away 1/2 x 1/3 x 1/4 = 1/24.

7. The average of all natural numbers from 1 to 2 000 (1 000.5) multiplied by 2 000 will yield the sum sought. This sum is equal to (1 000.5 x 2 000) 2 001 000.

8. Only one block has only one face that is covered with glue, the one with the dot. Eight blocks have at least two faces that are covered with glue.

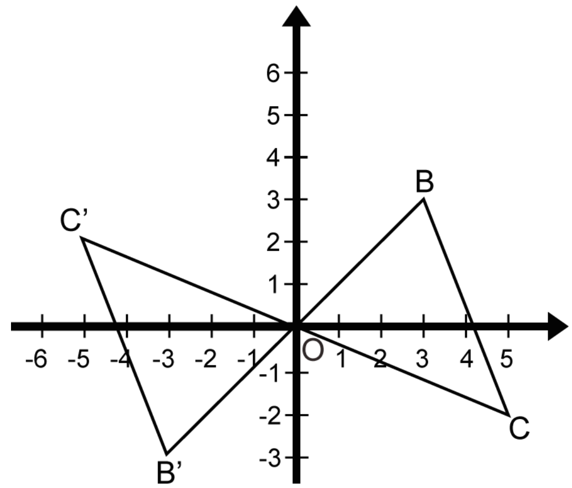


9. If half of N is 12, N is 24 and 4N is equal to 96.

10. If c is the length of the side of the square, its area is c^2. If the length of each side were reduced by 25%, each side of the new square would have a length of 3/4 c. Then the area of the square would be 9/16 c^2. Thus the area of the square would be reduced by (c^2 - 9/16 c^2) 7/16 c^2. The area would be reduced by (7/16) or 43.75%.

11. The LCM (3, 7) is 21. The GCD (12, 18) is 6. The product of 21 x 6 is 126.

12. Rotate Δ OBC 180º about the origin O. The coordinates of B’ (image of B) are (-3, -3).



13. Mathusalem has lost 40% of his weight during the summer. His weight at the beginning of the summer was (100 ÷ 60 x 100) 166 2/3 kg. Rounded to the nearest kg, his weight at the beginning of the summer was 167 kg.

14. If 1/2 + 1/3 + 1/n = 53/6, then 1/n = 53/6 - 5/6 = 8 and n = 1/8.

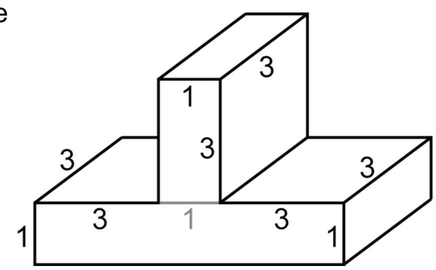
15. The probability that she will choose the yellow sweater and the wool skirt is (1/4 x 1/2) 1/8.

16. With one digit, you can form 1 (1^1) natural number, with 2 digits, you can form 4 (2^2) natural numbers, with 3 digits, you can form 3^3 or (27) twenty-seven 3-digit natural numbers.

17. If P = $10 + 10^2 + 10^3 + 10^4 + 10^5$, the sum of P’s digits (111 110) is 5.

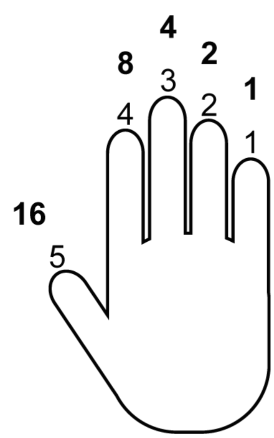
18. The algebraic expression that can generate the sequence of numbers that yield a remainder of 2 when divided by 4 (2, 6, 10, 14, …) is 4n + 2.

19. The volume, in cm^3, of the rectangular solid shown in the diagram is equal to the sum of the volumes of the two rectangular prisms that make up the solid. This volume is equal to (1 x 3 x 3 + 7 x 3 x 1) 30 cm^3.



20. If x = -2, the value of $-3x + 2x^2 - 2x^3$ is $(-3(-2) + 2(-2)^2 - 2(-2)^3 = 6 + 8 + 16)$ 30.

21. Binary Ben uses the binary system. Finger 1 represents the number 1 (2^0) when it is straight and zero when it is curled. Finger 2 represents the number 2 (2^1) when it is straight and zero when it is curled. When finger 3 is straight, it represents 4 (2^2). When finger 4 is straight it is equal to 8 (2^3), and when finger 5 is straight, it is equal to 16 or 2^4. When all five fingers are straight, Ben represents the number (1 + 2 + 4 + 8 + 16) 31. To represent the number 7, he must hold fingers 1-2-3 straight. By the way, Binary Ben can represent a total of 32 natural numbers(the numbers from 0 to 31).



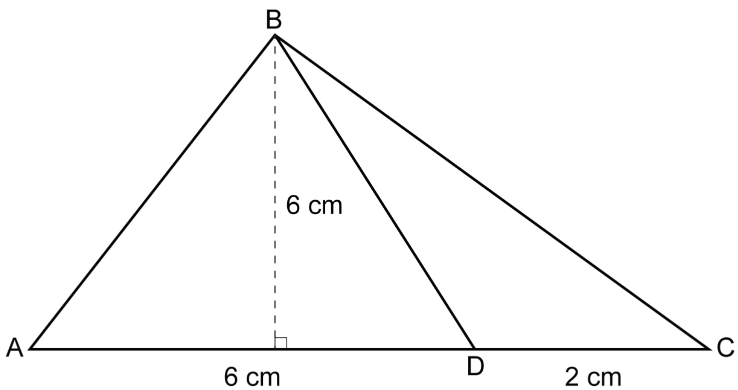
22. The value of A (see figure below) is 1 because if A had a value of 2 or more, the product of the multiplication would give a 5-digit number. Letter B cannot be equal to 0, 2, 4, 6, or 8 because the unit digit of DEDB would be zero. B must be equal to 5 because it is the only odd digit that will yield a unit digit of 5 in the result DEDB. After additional calculations and deductions, it is easy to show that C is equal to 3 and D is equal to 7.

23. The prime factors of 210 are {2, 3, 5, 7}. The number whose 3 digits yield a product of 210 must be composed of 3 digits which have the values 6 (2 x 3), 5, and 7. Even if there are many numbers that are composed of the same 3 digits (567, 756, 657, …), their sum is always (5 + 6 + 7) 18.

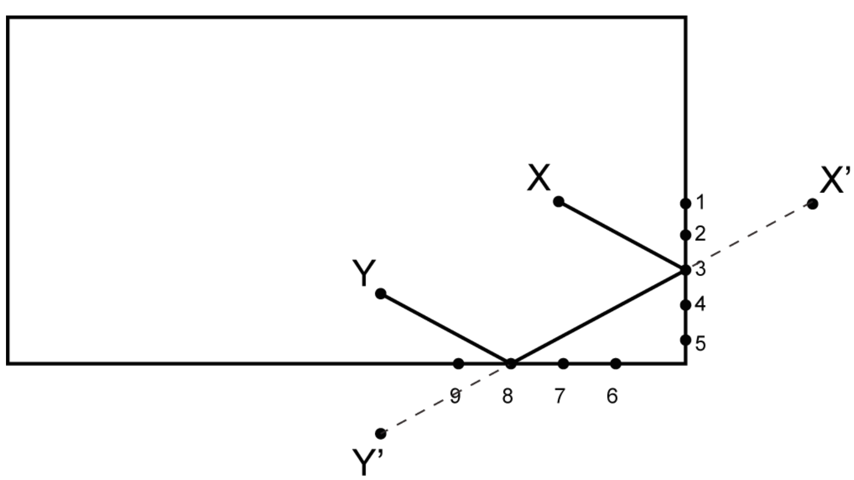
$$\begin{array}{r} A\ B\ C\ B \\ \times \qquad\quad 5 \\ \hline D\ E\ D\ B \end{array}$$

24. If P = p_1 x p_2 x p_3 x p_4 x … p_{50}, then 10 (2 x 5) and 30 (2 x 3 x 5) are factors of P.

25. The base AC of Δ ABC is equal to (24 cm^2 x 2 ÷ 6 cm) 8 cm. Given that AD : DC = 3, we find that AD = 6 cm and DC = 2 cm. The area of Δ BDC is equal to (2 cm x 6 cm ÷ 2) 6 cm^2.



26. The shortest path between point X and point Y is X-3-8-Y. This path is the shortest because the shortest distance between two points of a plane is always a straight line. Let us explain! The diagram shows point X and its image, point X’ (as if the wall were a mirror), and point Y and its image, point Y’. The shortest distance between points X’ and Y’ is surely X’Y’ (the straight line between X’ and Y’). Note that the path X-3-8-Y has the same length as the virtual path X’-3-8-Y’. Indeed, the distance X’-3 is equal to the distance X-3 because the wall is an axis of symmetry. We can apply the same logic to the distances 8-Y and 8-Y’. Mathilda should follow the path X-3-8-Y if she wants to increase her chances of winning the race.



How would you find the shortest path between X and Y (figure hereafter) if from X the students first must run to a point of wall BC, then to a point of wall CD, and finally, to a point of wall DA before going to point Y?