Mathematica Centrum

Together, let's shape the mathematicians of the future

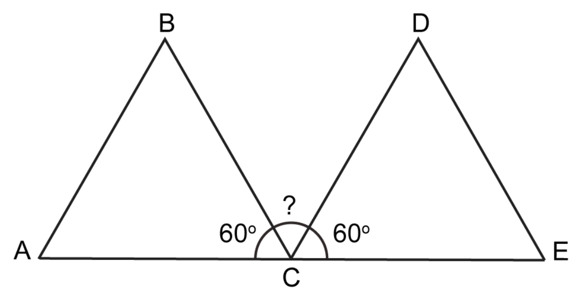
LAGRANGE PREPARATORY TEST 2017
DETAILED SOLUTIONS

1. 5 = 2 + 3, 8 = 3 + 5, 9 = 2 + 7, 12 = 5 + 7, and 24 = 11 + 13. They can all be written as the sum of two prime numbers.

2. The number 20 has 6 divisors (1, 2, 4, 5, 10, 20). The number 10 has 4 divisors (1, 2, 5, 10).

3. Today is Wednesday. In 85 days, it will be (85 = 12 x 7 + 1) a Thursday.

4. ABC and CDE are two equilateral triangles. A, C, and E are three points of line segment AE. Angle ACB + angle BCD + angle DCE = 180º. 60º + angle BCD + 60º = 180º. Angle BCD is equal to (180º - 120º) 60º.



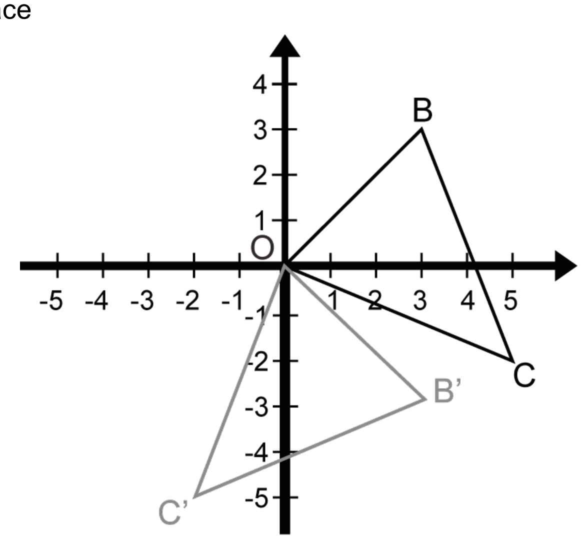
5. Mathilda printed 100 consecutive integers. If the largest of these integers is 40, then the smallest is -59. Do not forget the 0. There are 40 integers greater than 0 and (100 - 41) 59 smaller than 0.

6. 120 = 2 x 2 x 2 x 3 x 5. Three prime numbers between 1 and 10 (2, 3, and 5) are factors of 120.

7. Andrea has a 3 000 cm^2 painting. She is going to replace it by a 4 000 cm^2 painting. The area will increase by (1 000 cm^2 ÷ 3 000 cm^2 x 100) 33 1/3%.

8. The average of 1/2 and 1/6 is ((1/2 + 1/6) ÷ 2) 1/3. If 1/n = 1/3, the value of n is 3 and the value of 4n is 12.

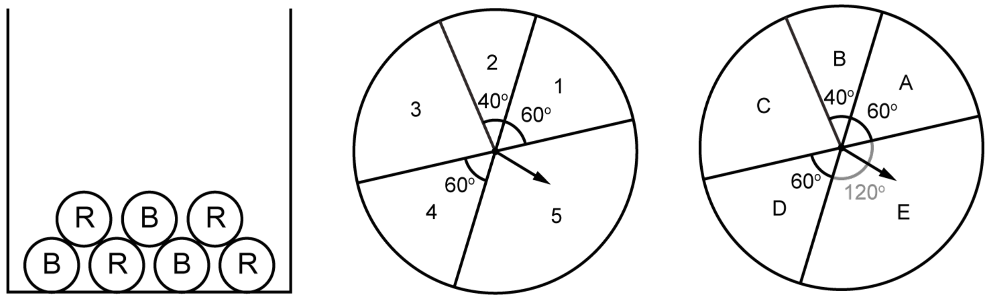
9. Rotate ∆ OBC 90º clockwise about the origin O. The coordinates of B' (image of B) are (3, -3).



10. Mathusalem has chosen three different integers from the following set: {-4, -3, -1, 0, 3, 4}. The smallest possible product of the three integers chosen is (-4 x 3 x 4) -48.

11. 1 dm^2 = 1 dm x 1 dm = 10 cm x 10 cm = 100 cm^2 and 10 dm^2 = 10 x 100 cm^2 = 1 000 cm^2.

12. The probability that she will get a red ball is 4/7, the probability of getting an even number is (40º + 60º = 100º) 100º ÷ 360º or 5/18, and the probability of getting a vowel (A or E) is 180º ÷ 360º or 1/2. The probability of getting a red ball, an even number, and a vowel is (4/7 x 5/18 x 1/2) 5/63.

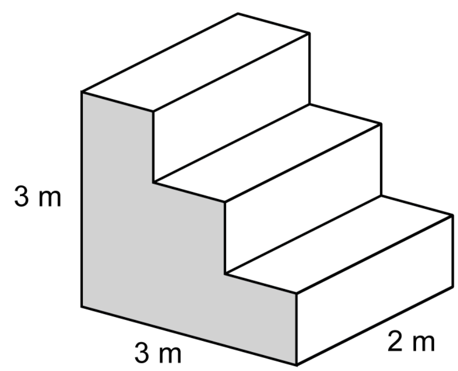


13. If N and M are two positive integers and N^2 = 3M, then 3M is a square number and a multiple of 3. The numbers 9, 36, and 81 are the only square numbers less than 100 that are multiples of 3. If N^2 = 9, N = 3 and M = 3, and N + M = 6. If N^2 = 36, N = 6 and M = 12, and N + M = 18. If N^2 = 81, N = 9 and M = 27, and N + M = 36. Only 6 and 18 can represent the sum of N + M.

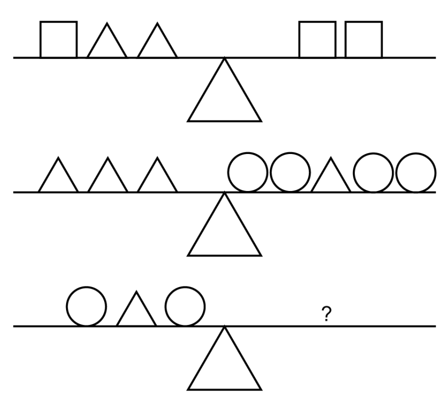
14. In 20 years, Mathusalem will be twice as old as he was 20 years ago. Forty years will go by between 20 years ago and 20 years from now. If Mathusalem will be twice as old, this means that he was 40 years old 20 years ago and will be 80 years old 20 years from now. He is now 60 years old and will be 70 years old 10 years from now.

15. There are 8 teams in a tournament. Each team will play every other team twice. This means that each team will play 7 other teams twice. Each team will play 7 x 2 games. Eight teams will play 8 x 7 x 2 games. However each game is played by 2 teams; therefore (8 x 7 x 2 ÷ 2) only 56 games need to be scheduled for the tournament.

16. The equation 10^6 x 10^n = 1 000^4 can be written as 10^6 x 10^n = 1 000 x 1 000 x 1 000 x 1 000 = 10^{12}. The value of n is 6.

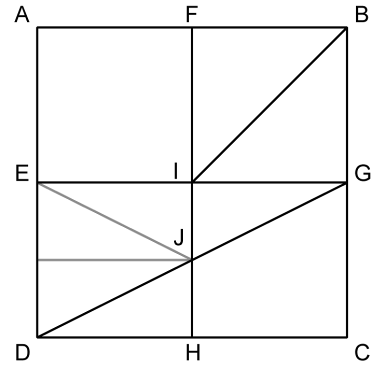


17. The area of the prism's bottom is (3 m x 2 m) 6 m^2. The area of the prism's top (the top of the 3 steps) is also 6m^2. The area of the prism's two sides (the shaded surface) is (2 x 6 m^2) 12 m^2. The back of the prism is (3 m x 2 m) 6 m^2. The front of the prism (the front of the 3 steps) is also 6 m^2. The total area of the surface of the prism is 36 m^2.



18. From the first diagram, we can see that one square weighs as much as two triangles. From the second diagram, we can see that one triangle weighs as much as two circles. One square weighs as much as four circles. It will take 1 square weight to balance the weights that are on the left side of the third diagram because the left side of the diagram weighs as much as 4 circles.

19. Square ABCD is divided into 4 smaller squares. The area of trapezium EIJD is equal to 3/4 of one of these smaller squares. Trapezium ABIE is equal to 1 1/2 of these smaller squares. The area of trapezium EIJD is (3/4 ÷ 1 1/2) 1/2 of the area of trapezium ABIE?

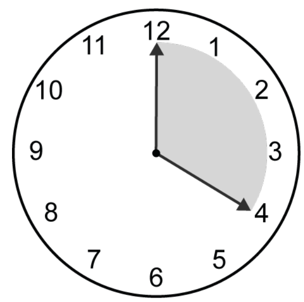


20. Let us suppose that the price of the shirt was 100. The price with the discount was 80 and the price before the profit of 20% was (80 ÷ 1.2) 66 2/3. His profit, if he had not given the discount would have been (100 - 66 2/3) 33 1/3. This represents a (33 1/3 ÷ 66 2/3 x 100) 50% profit.

21. There are between 20 and 40 identical marbles in a bag. When I count the marbles by groups of 4, I have 3 left. This suggests that the number of marbles can be represented by the algebraic expression 4n + 3. The number of marbles could be 3, 7, 11, 15, 19, 23, 27, 31, 35, … . When I count them by groups of 5, I have 2 left. This suggests that the number of marbles can be represented by the algebraic expression 5n + 2. The number of marbles could be 2, 7, 12, 17, 22, 27, 32, 37, … . There were 27 marbles in the bag.

22. 500 g of this solution contains 300 g of water. Let x be the quantity of water she must add to the solution. Then (300 + x)/ 500 + x = 0.8. This equation becomes 300 + x = 400 + 0.2x. We find x = 500. She must add 500 g of water to the 500 g of the original solution.

23. The minute hand of a clock measures 12 cm. In 20 minutes, it will cover an area of (1/3 ($\pi 12^2$)) 48π cm^2.



24. The positive integer values of x and y when 2x + 3y = 40 are x = 2 and y = 12, x = 5 and y = 10, x = 8 and y = 8, … , x = 17 and y = 2. Replacing these values in the expression x + 4y, we find that the maximum value of x + 4y is 50. Indeed, when x = 2 and y = 12, the value of 1x + 4y is (1(2) + 4(12)) 50.