

Mathematica Centrum

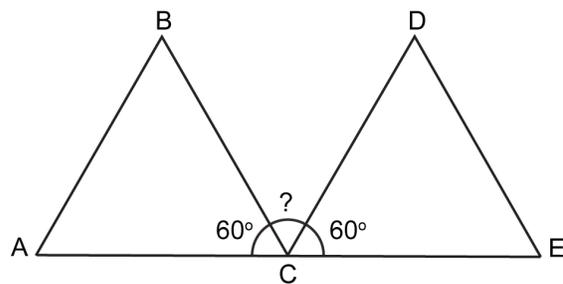
Together, let's shape the mathematicians of the future

NEWTON PREPARATORY TEST 2017 DETAILED SOLUTIONS

1. $5 = 2 + 3$, $8 = 3 + 5$, $9 = 2 + 7$, $12 = 5 + 7$, and $24 = 11 + 13$. They can all be written as the sum of two prime numbers.
2. The number 20 has 6 divisors (1, 2, 4, 5, 10, 20).
The number 10 has 4 divisors (1, 2, 5, 10).

3. Today is Wednesday. In 85 days, it will be ($85 = 12 \times 7 + 1$) a Thursday.

4. ABC and CDE are two equilateral triangles.
A, C, and E are three points of line segment AE.
Angle ACB + angle BCD + angle DCE = 180° .
 $60^\circ + \text{angle BCD} + 60^\circ = 180^\circ$. Angle BCD is equal to $(180^\circ - 120^\circ) 60^\circ$.



5. Mathilda printed 100 consecutive integers. If the largest of these integers is 40, then the smallest is -59. Do not forget the 0. There are 40 integers greater than 0 and $(100 - 41) 59$ smaller than 0.
6. $120 = 2 \times 2 \times 2 \times 3 \times 5$. Three prime numbers between 1 and 10 (2, 3, and 5) are factors of 120.

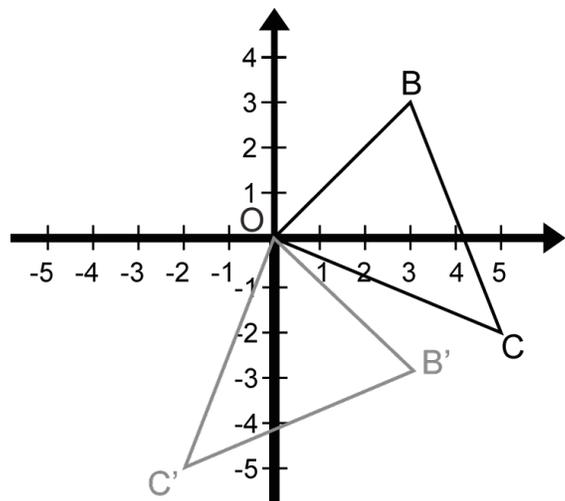
7. Andrea has a $3\,000\text{ cm}^2$ painting. She is going to replace it by a $4\,000\text{ cm}^2$ painting. The area will increase by $(1\,000\text{ cm}^2 \div 3\,000\text{ cm}^2 \times 100) 33\frac{1}{3}\%$.

8. The average of $\frac{1}{2}$ and $\frac{1}{6}$ is $(\frac{1}{2} + \frac{1}{6}) \div 2 = \frac{1}{3}$. If $\frac{1}{n} = \frac{1}{3}$, the value of n is 3 and the value of $4n$ is 12.

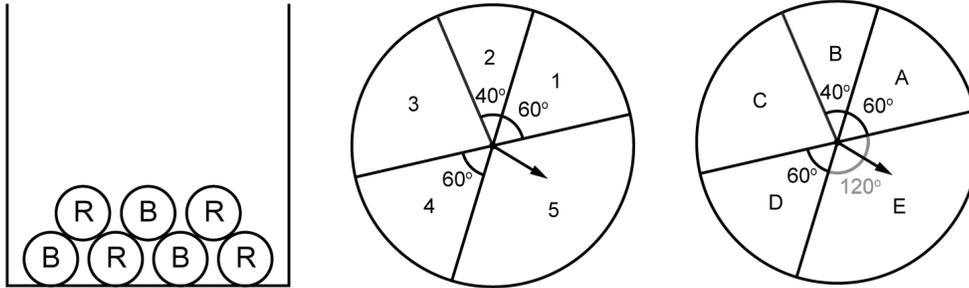
9. Rotate $\triangle OBC$ 90° clockwise about the origin O.
The coordinates of B' (image of B) are (3, -3).

10. Matusalem has chosen three different integers from the following set: $\{-4, -3, -1, 0, 3, 4\}$. The smallest possible product of the three integers chosen is $(-4 \times 3 \times 4) -48$.

11. $1\text{ dm}^2 = 1\text{ dm} \times 1\text{ dm} = 10\text{ cm} \times 10\text{ cm} = 100\text{ cm}^2$
and $10\text{ dm}^2 = 10 \times 100\text{ cm}^2 = 1\,000\text{ cm}^2$.



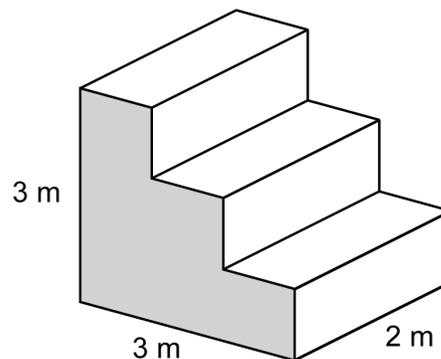
12. The probability that she will get a red ball is $\frac{4}{7}$, the probability of getting an even number is $(40^\circ + 60^\circ) \div 360^\circ$ or $\frac{5}{18}$, and the probability of getting a vowel (A or E) is $180^\circ \div 360^\circ$ or $\frac{1}{2}$. The probability of getting a red ball, an even number, and a vowel is $(\frac{4}{7} \times \frac{5}{18} \times \frac{1}{2}) \frac{5}{63}$.



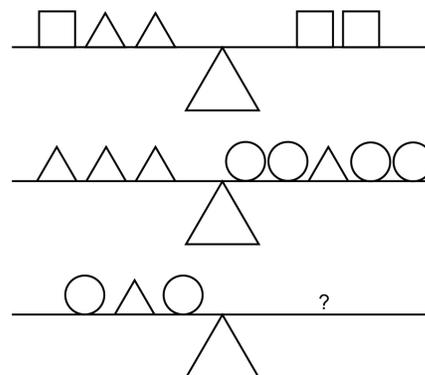
13. If N and M are two positive integers and $N^2 = 3M$, then $3M$ is a square number and a multiple of 3. The numbers 9, 36, and 81 are the only square numbers less than 100 that are multiples of 3. If $N^2 = 9$, $N = 3$ and $M = 3$, and $N + M = 6$. If $N^2 = 36$, $N = 6$ and $M = 12$, and $N + M = 18$. If $N^2 = 81$, $N = 9$ and $M = 27$, and $N + M = 36$. Only 6 and 18 can represent the sum of $N + M$.
14. In 20 years, Mathusalem will be twice as old as he was 20 years ago. Forty years will go by between 20 years ago and 20 years from now. If Mathusalem will be twice as old, this means that he was 40 years old 20 years ago and will be 80 years old 20 years from now. He is now 60 years old and will be 70 years old 10 years from now.
15. There are 8 teams in a tournament. Each team will play every other team twice. This means that each team will play 7 other teams twice. Each team will play 7×2 games. Eight teams will play $8 \times 7 \times 2$ games. However each game is played by 2 teams; therefore $(8 \times 7 \times 2 \div 2)$ only 56 games need to be scheduled for the tournament.

16. The equation $10^6 \times 10^n = 1\,000^4$ can be written as $10^6 \times 10^n = 1\,000 \times 1\,000 \times 1\,000 \times 1\,000 = 10^{12}$. The value of n is 6.

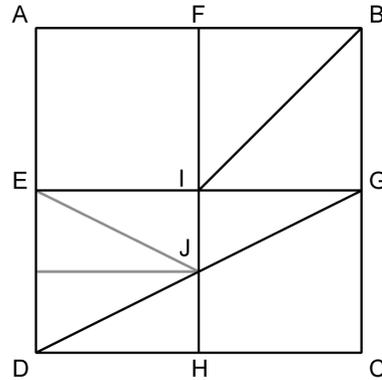
17. The area of the prism's bottom is $(3\text{ m} \times 2\text{ m}) 6\text{ m}^2$. The area of the prism's top (the top of the 3 steps) is also 6 m^2 . The area of the prism's two sides (the shaded surface) is $(2 \times 6\text{ m}^2) 12\text{ m}^2$. The back of the prism is $(3\text{ m} \times 2\text{ m}) 6\text{ m}^2$. The front of the prism (the front of the 3 steps) is also 6 m^2 . The total area of the surface of the prism is 36 m^2 .



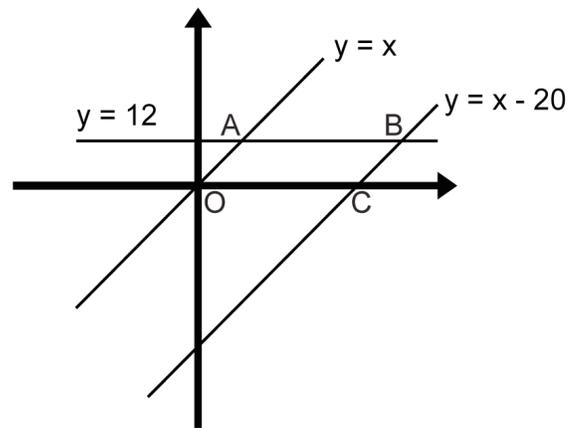
18. From the first diagram, we can see that one square weighs as much as two triangles. From the second diagram, we can see that one triangle weighs as much as two circles. One square weighs as much as four circles. It will take 1 square weight to balance the weights that are on the left side of the third diagram because the left side of the diagram weighs as much as 4 circles.



19. Square ABCD is divided into 4 smaller squares. The area of trapezium EIJD is equal to $\frac{3}{4}$ of one of these smaller squares. Trapezium ABIE is equal to $1\frac{1}{2}$ of these smaller squares. The area of trapezium EIJD is $(\frac{3}{4} \div 1\frac{1}{2}) \frac{1}{2}$ of the area of trapezium ABIE?
20. Let us suppose that the price of the shirt was 100. The price with the discount was 80 and the price before the profit of 20% was $(80 \div 1.2) 66\frac{2}{3}$. His profit, if he had not given the discount would have been $(100 - 66\frac{2}{3}) 33\frac{1}{3}$. This represents a $(33\frac{1}{3} \div 66\frac{2}{3} \times 100) 50\%$ profit.



21. There are between 20 and 40 identical marbles in a bag. When I count the marbles by groups of 4, I have 3 left. This suggests that the number of marbles can be represented by the algebraic expression $4n + 3$. The number of marbles could be 3, 7, 11, 15, 19, 23, 27, 31, 35, When I count them by groups of 5, I have 2 left. This suggests that the number of marbles can be represented by the algebraic expression $5n + 2$. The number of marbles could be 2, 7, 12, 17, 22, 27, 32, 37, There were 27 marbles in the bag.



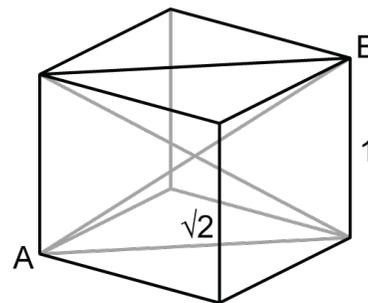
22. The coordinates of point C are $(x, 0)$. The value of x is given by the equation $0 = x - 20$. We find $x = 20$. The length of line segment OC is 20. Lines $y = x$ and $y = x - 20$ are parallel (they have the same slope). Polygon ABCO is a parallelogram and $OA = CB$. The coordinates of point A are found by solving the system of equations $y = x$ and $y = 12$. The coordinates of point A are $(12, 12)$. To find OA, we must solve equation $OA^2 = 12^2 + 12^2$. We find $OA = 12\sqrt{2}$. The perimeter of polygon ABCO is $(2 \times 20 + 2 \times 12\sqrt{2}) 40 + 24\sqrt{2}$.

$$A = \{3, 4, 5, 6, 7\}$$

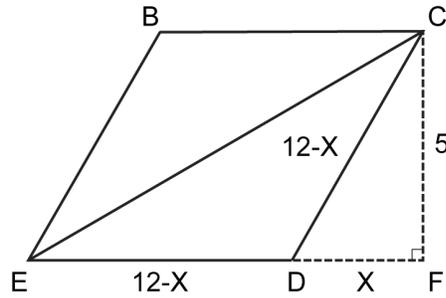
$$B = \{8, 9, 10, 11\}$$

23. There are $(5 \times 4) 20$ possible pairs of numbers that can be formed by choosing two numbers from A and B. There are 3 sums that are less than 13: $3 + 8$, $3 + 9$, and $4 + 8$. The probability that the sum of the two numbers is less than 13 is $\frac{3}{20}$.

24. The greatest distance between two vertices of a cube is the length of the "diagonal" AB of the cube. This distance is equal to $\sqrt{3}$ u. We find it by solving the equation $AB^2 = (\sqrt{2})^2 + (1)^2$. The cube has four such "diagonals". These diagonals intersect at their mid-points at a point that is the centre of the cube.



25. Let X be the value of line segment DF . Then $ED = DC = 12 - X$. Using the Pythagorean theorem, we can write that $(12 - X)^2 = X^2 + 5^2$. This equation can be written as $(12 - X)(12 - X) = X^2 + 25$. The left side of this equation can be expanded (see diagram below) and written as $144 - 12X - 12X + X^2$. Solving for X , we find $X = 119/24$. The length of one side $(12 - X)$ of rhombus $BCDE$ is closest in value to $(12 - 119/24)$ 7 cm.



$$(12-X)(12-X) = 144 - 12X - 12X + X^2$$