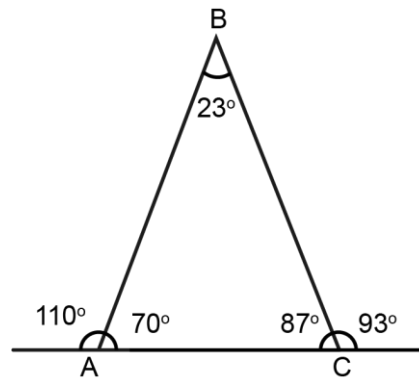


Mathematica Centrum

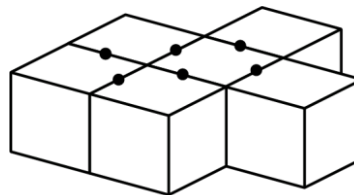
Together, let's shape the mathematicians of the future

EULER PREPARATORY TEST 2018 DETAILED SOLUTIONS

- The numbers 3 ($3 + 1 = 4$), 15 ($15 + 1 = 16$), and 48 ($1 + 48 = 49$) yield a perfect square when you add 1 to them.
- The largest possible sum, less than 10, of two consecutive prime numbers is $(3 + 5) 8$.
- If $\frac{3}{4}$ of a number is equal to 8, then $\frac{9}{4}$ of the same number is equal to $(8 \times 3) 24$.
- $n \div \frac{1}{6} = 18$ is equivalent to $n \times 6 = 18$.
The value of $n \times 2$ is $(3 \times 2) 6$.
- The value of angle B is 23° .
- $(50\% \text{ of } 50\%)\%$ is equal to $(\frac{1}{2} \times \frac{1}{2} = \frac{1}{4})$
 $\frac{1}{4} \%$ or $\frac{1}{400}$.

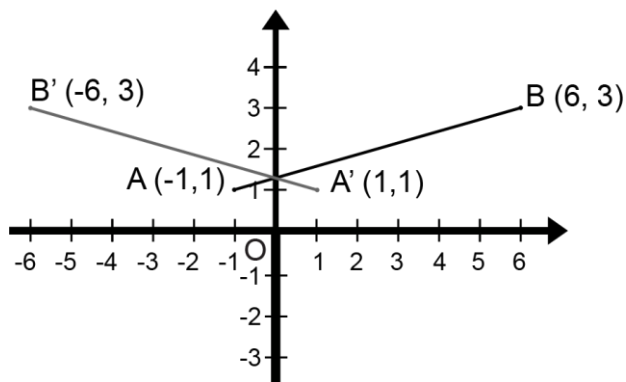


- Starting with -9, all integers are written in increasing order: -9, -8, -7, -6, -5, -4, -3, -2, The 20th digit that will be written is a (-9, -8, -7, ... -1, 0, 1, 2, 3, ... 9, 10) 1.
- When the bottom layer of blocks are glued together, 12 faces will have glue on them. Another six faces will have glue on them when the two top blocks are put into place. In all, 18 faces of the eight blocks have glue on them.
- The price after the 40% increase will be $(\$100 + \$40) \$140$. The price after the 30% decrease will be $(\$150 - \$45) \$105$. When bought together, the price of the two items will be $(\$140 + \$105) \$245$. The price of the two items decreases by $(\$250 - \$245 = \$5$ and $(\$5 / \$250) \times 100 = 2/100) 2\%$
- If $P + Q = 12$, the greatest possible value of the expression $P \times Q$ is 36.



P	Q	P X Q
0	12	0
1	11	11
2	10	20
3	9	27
4	8	32
5	7	35
6	6	36

11. Line segment AB is reflected in the y-axis. The coordinates of the images of points A and B, after the reflection, are respectively, (1, 1) and (-6, 3).

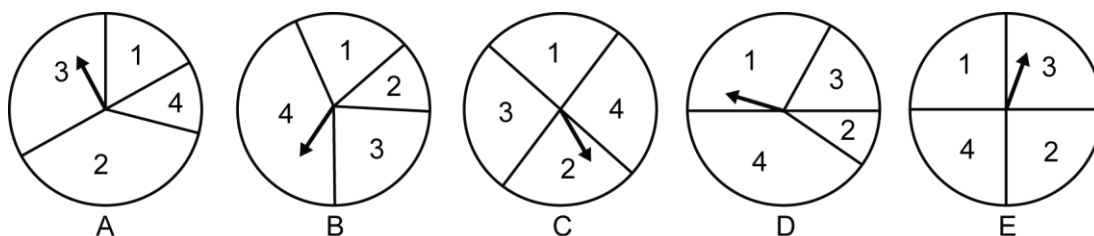


12. The result of the number of faces of a cube (6) plus the number of edges of a cube (12) plus the number of vertices of a cube (8) minus the number of angles of a cube (24) is equal to 2.

13. The LCM (3, 4, 5) = 60.

14. $1 \text{ cm}^2 = 100 \text{ mm}^2$ and $20 \text{ cm}^2 = 20 \times 100 \text{ mm}^2 = 2\,000 \text{ mm}^2$

15. The probability of getting a 2 or a 4 with spinner B is more than $1/2$.



16. There are 16 (1×1 squares), 9 (2×2 squares), 4 (3×3 squares), and 1 (4×4 square) in a 4×4 square (fig.1). In all, there are $(16 + 9 + 4 + 1)$ 30 squares. In other words, there are $(4^2 + 3^2 + 2^2 + 1^2)$ distinct squares in a 4×4 square. In a 3×3 square, you can count a total of $(3^2 + 2^2 + 1^2)$ 13 distinct squares. Verify this assertion by counting the number of squares in the 3×3 square shown in the diagram (fig.2).

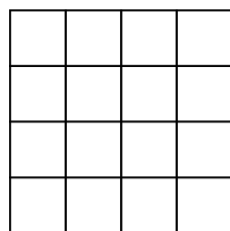


fig.1

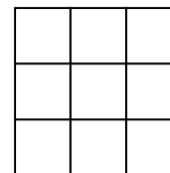


fig.2

17. The area of the small triangle can be written as $(b \times h)/2$. The area of the large triangle can be written as $(2b \times 2h)/2$. The area of the small triangle is $((b \times h)/2) \div 2 / (2b \times 2h)$ the area of the large triangle. The area of the small triangle is $1/4$ of the area of the large triangle.

18. The sum of angle A + angle B + angle C = 180° .
 Angle C + 20° + angle C + 10° + angle C = 180° .
 We find that angle C = 50° and angle B = 60° .
 The sum of angle B + angle C = $50^\circ + 60^\circ = 110^\circ$.

$\angle A = \angle C + 20^\circ$
$\angle B = \angle C + 10^\circ$
$\angle C = \angle C$

19. The only 3 prime numbers that add up to 19 are 3, 5, and 11. One of these prime numbers must be 11.

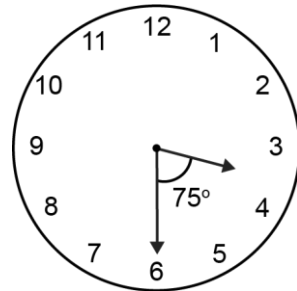
20. If the area of a square is 4, the length of each side is 2. If each side is quadrupled, the area of the new square is (8×8) 64.

21. If $N + M = 4$, we find 5 possible pairs of values of N and M for which the number **8NM51** is divisible by 9. The 5-digit numbers are 80451, 81351, 82251, 83151, and 84051.

N	M
0	4
1	3
2	2
3	1
4	0

22. At 3:30, the hour hand and the minute hand form an angle of 75° . Indeed, between 3:00 and 3:30, the minute hand moves $(30^\circ \times 6)$ 180° and the hour hand moves $(30^\circ \div 2)$ 15° , because there are 30° between two consecutive hours. At 3:30, the hour hand and the minute hand form an angle of $(2 \times 30^\circ + 15^\circ)$ 75° .

23. If $1 + 3 = 4$, $1 + 3 + 5 = 9$, and $1 + 3 + 5 + 7 = 16$, then $1 + 3 + 5 + 7 + 9 + 11 + \dots + 21 = 121$. To find the sum of these consecutive odd number series (starting with 1), you must find the square of the number of terms in the series. The number of terms in the series $(1 + 3)$ is 2 and $2^2 = 4$. The number of terms in the series $(1 + 3 + 5 + 7)$ is 4 and $4^2 = 16$. The number of terms in the series $(1 + 3 + 5 + 7 + 9 + \dots + 21)$ is $(21 + 1 = 22, 22 \div 2 = 11, \text{ and } 11^2 = 121)$ 121.



24. A printer must print all the natural numbers between 0 and 100. It must print all the digits of the numbers from 1 to 99. From 1 to 9 there are 9 digits. From 10 to 99 there are a total of (90×2) 180 digits. At 3 digits per second, it will take $(180 + 9 = 189 \text{ and } 189 \div 3 = 63)$ 63 seconds to print all these numbers.