Mathematica Centrum
Together, let's shape the mathematicians of the future

NEWTON PREPARATORY TEST 2019
DETAILED SOLUTIONS

1. The average value of an angle in a triangle is (180° ÷ 3) 60°. The largest angle in a triangle is larger or equal to 60°. The smallest angle in a triangle is smaller or equal to 60°.

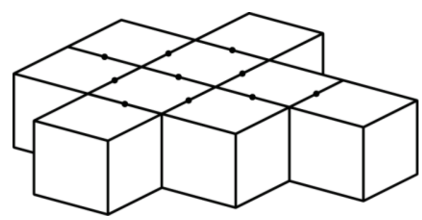
2. The largest prime factor of 777 is 37.

3. 200% x 1/2 -(-1 + 5) = 2 x 1/2 -(4) = -3.

4. The product of two natural numbers is 20 (20 x 1, 10 x 2, …). The largest possible sum of these two numbers is (20 + 1) 21.

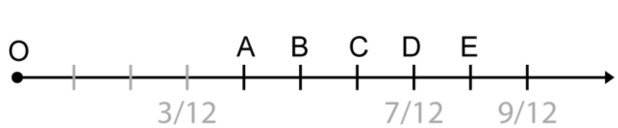
5. 2! = 1 x 2 = 2, 3! = 1 x 2 x 3 = 6, 3!! = (3!)! = 6! = 720. The expression 2!!!! is the smallest (2!!!! = 2!!! = 2!! = 2! = 2).

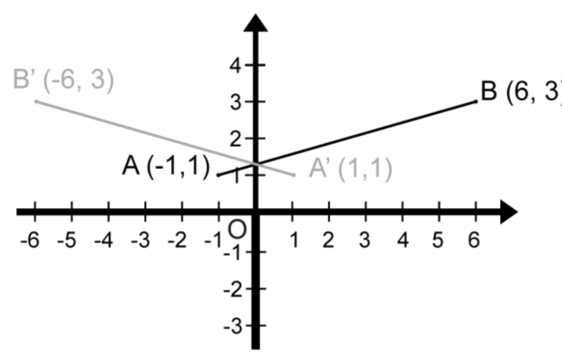
6. The 9 blocks shown in the diagram have 20 glued faces. In all, the eleven blocks have (20 + 6) 26 glued faces.



7. The average of 5 different integers smaller than 0 is -5. The sum of these 5 integers is (5 x -5) -25. You can find many different sets of 5 negative integers whose average is -5, but there is one set where the smallest integer is the smallest possible. This integer is (-25 - (-1 + -2 + -3 + -4)) -15. Go to the end of the document for further explanations.

8. The fractions 1/4 (3/12) and 3/4 (9/12) are represented on the number line below. If the origin of the number line is 0, the letter that represents the fraction 7/12 is D.





9. Line segment AB is reflected in the y-axis. The coordinates of the images of points A and B, after the reflection, are respectively (1, 1) and (-6, 3).

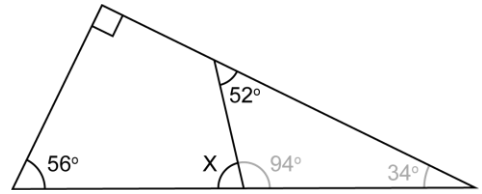
10. 10% of 20 (2) + 20% of 50 (10) + 30% of 30 (9) = 21.

11. From any vertex, you can draw (10 - 3) 7 diagonals. From the 10 vertices, you can draw (10 x 7) 70 diagonals. However, each diagonal being shared by 2 vertices, the number of diagonals that can be drawn is (10 x 7 ÷ 2) 35.

12. 1 dm^3 = 1 dm x 1 dm x 1 dm = 10 cm x 10 cm x 10 cm = 1 000 cm^3 and 10 dm^3 = 10 000 cm^3.

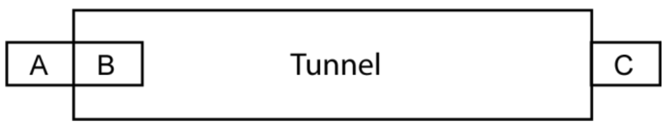
13. A car performs a sequence of five displacements: N2, W4, S6, E4, N1. The (W4) 4 units to the west cancel out the (E4) 4 units to the east. The 3 displacements (N2 + N1 + S6) are equivalent to the displacement S3.

14. The product of all the prime numbers smaller than 10 (2 x 3 x 5 x 7) is not divisible by 9.



15. The measure of angle X is (180° - 94°) 86°.

16. A train moving at 120 km/h takes 15 s to completely enter a tunnel (to pass from position A to position B). In 15 s (1/4 minute), the train covers a distance equal to its own length. At a speed of (120 km/60 min) 2 km/min, the train covers a distance of (2 km x 1/4 min) 0.5 km.

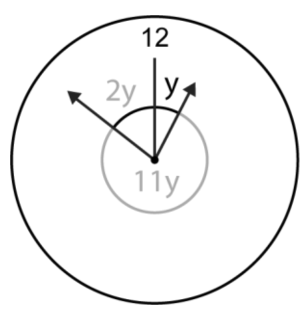


17. The LCM of 6, 8, and 10 is (2 x 2 x 2 x 3 x 5) 120.

18. If x = -3, the value of x^2 - 5x is $(-3)^2$ - 5(-3) = 24.

19. The maximum number of Sundays that can occur in a period of 60 days is 9. This occurs if the first day of the 60 day period is a Sunday. The sequence 1, 8, 15, 22, 29, 36, 43, 50, **57**, … tells us that the last Sunday of the period occurs on the 57th day. However, finding the maximum number of Sundays in a longer period of time could be difficult. Writing the sequence above as an algebraic expression can make things easier. The algebraic expression that defines this sequence is 1 + 7n. When n = 0, the value of the expression is 1 (the first Sunday of the period). When n = 1, the value is 8 (the second Sunday of the period). What is the value of n that gives the last Sunday of the period? The equation 1 + 7n = 60 will yield this value. We find a value of n close to 8.43. We know hat n must be an integer and this integer cannot be 9 (the value of the expression would be 64) so it must be 8. When n = 8, the expression tells us that the last Sunday of the period occurs on the 57th day. The maximum number of Sundays for any number of days is given by n + 1, because the first Sunday is given by n = 0. The maximum number of Sundays in a period of 60 days is (n + 1) 9.

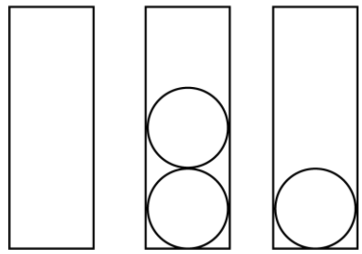
20. Any term in the sequence (except for the first one) is equal to 4 times the preceding term minus 1. Indeed 3 = 4 x 1 - 1, 11 = 4 x 3 - 1, … . The next term in the sequence: 1, 3, 11, 43, … is (4 x 43 - 1) 171.



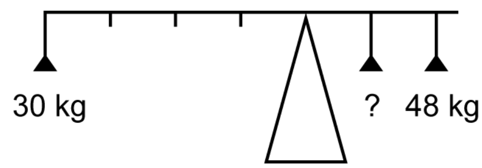
21. If you could spend $2 every 20 seconds, you could spend 3 600 times more in 20 hours. In 20 hours, you could spend (3 600 x $2) $7 200.

22. There is a time on a clock between 12:00 and 1:00, when angle x is equal to twice angle y. At 12:00, the two hands of the clock are together. When the hour hand moves z°, the minute hand moves 12z°. When the hour hand moves y°, the minute hand moves 12y°. Now look at the diagram. The sum of y + 11y + 2y equals 360°. We find a value close to 25.7°. The value which is closest to the value of angle y is 26°.

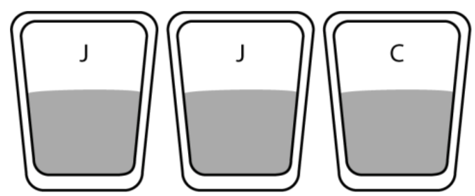
23. You can put 1, 2, or 3 balls in the third box. With 1 ball in the third box, we can find 3 ways: 0-2-1, 2-0-1, and 1-1-1. With 2 balls in the third box, we can find 2 ways: 1-0-2 and 0-1-2. With 3 balls, we can find one way: 0-0-3. In all, there are 6 different ways to put 3 balls in the 3 boxes, if the third one must have at least one.



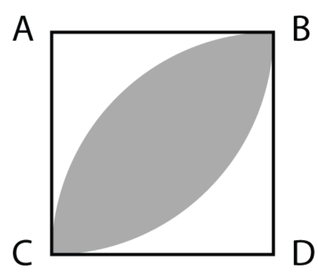
24. To explain this problem, we must talk about the concept of torque or moment of a force. Archimedes used this concept to explain rotational motion in levers. Archimedes defined a torque as the product of a force x distance. He defined distance as the distance from the force to the fulcrum. The distance of the 30 kg force to the fulcrum is 4. The torque or moment of the 30 kg force is 30 x 4 or 120. The torque of the 48 kg force is 48 x 2 or 96. However the 30 kg force causes a counter-clockwise rotation and the 48 kg force causes a clockwise rotation. There is equilibrium only when the counter-clockwise torques equal the clockwise torques. We need another force on the right side of the fulcrum to produce this equilibrium. The weight that will enable the system to be in equilibrium is given by the equation: 120 = 96 + F x 1. We find F = 24 kg.



25. Two of the three glasses on the right contain juice; the other contains coffee. The probability that you will drink only coffee is (1/3 x 0) 0. The probability that you will drink some juice is (1 - 0) 1 or 3/3. Logically, we know that the answer is 1, because if the first glass we drink from contains coffee, the second we drink from will surely contain juice.

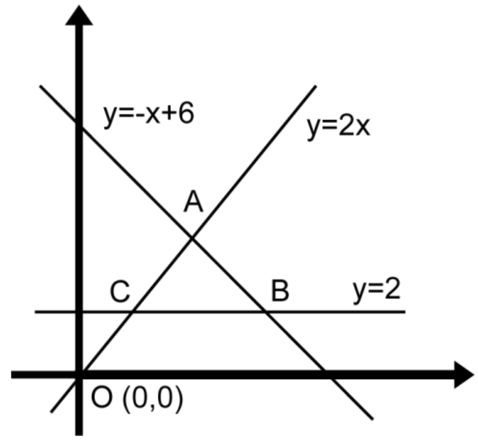


26. The diagram on the right shows a square whose side is 1 and also two arcs with centres at points A and D. Twice the area of the curved surface CBDC + the area of the shaded surface is equal to the area of the square. The area of the square is 1. The area of the curved surface is ($1 - \Pi 1^2/4$) $1 - \Pi/4$. From the equation $2(1 - \Pi/4)$ + the area of the shaded surface = 1, we find that the area of the shaded surface is equal to $\Pi/2 - 1$.

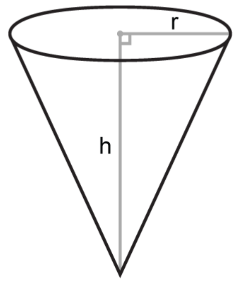


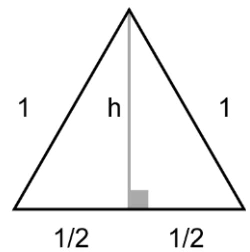
27. Mathilda can finish a job in 2 hours. Mathew can finish the same job in 3 hours. In 1 hour, Mathilda can finish 1/2 of the job. In 1 hour, Mathew can finish 1/3 of the job. Together, in 1 hour, they can finish (1/2 + 1/3) 5/6 of the job. They will take (1 ÷ 5 x 6) 6/5 h to finish the same job, if they work together.

28. Three straight lines, $y = 2x$, $y = -x + 6$, and $y = 2$ intersect at points A, B, and C. The coordinates of point C are found by solving the system of equations: $y = 2x$ and $y = 2$. We find $x = 1$ and $y = 2$. The coordinates of C are (1, 2). The coordinates of point A are found by solving the system: $y = 2x$ and $y = -x + 6$. We find $x = 2$ and $y = 4$. The coordinates of A are (2, 4). The distance between points A and C is given by the equation $d^2 = (2 - 1)^2 + (4 - 2)^2$. We find $d = \sqrt{5}$.



29. The volume of the cone below is $\Pi r^2h/3$.





30. The area of any triangle is b x h ÷ 2. The height h in the equilateral triangle is given by the equation: $h^2 + (1/2)^2 = 1^2$. We find $h = \sqrt{3}/2$. The area of the triangle is $(1 \times \sqrt{3}/2 \div 2)$ $\sqrt{3}/4$.

7. This question is problematic. Some students solve it using the method given on page 1. Others solve it simply by noting that the only choice possible is -15, because no set of 5 negative integers whose average is -5 can have the integers -16, -17, -18, or -19 as its smallest integer. Indeed, -15 is the smallest possible negative integer you can find when the other four are at their largest (-4, -3, -2, -1). This set is {-15, -4, -3, -2, -1} and its average is -5. Others cannot solve the problem because nothing in the question steers them in a direction that can help them solve the problem. The problem would otherwise be much clearer if the question were written differently. If the question was written as ''The smallest of these 5 integers could be" … . Because of the problematic nature of this problem, we have decided to cancel problem #16 (similar to this one) on the actual Euler, Lagrange, and Newton 2019 Contests. To be fair to every participant, to make sure that all students perform at their absolute best, and to insure, that perfect scores are still possible, we have decided to award a correct answer to all students, as long as they fill in the circle that has the letter B (16 B) on the response form. All contest managers will receive an important notice reminding them to kindly complete this task with their students participating in the Euler, Lagrange, and Newton Contests 2019.