Mathematica

Let's shape together the mathematicians of the future

PREPARATORY TEST 2008
DETAILED SOLUTIONS

EULER (7th) – LAGRANGE (8th) – NEWTON (9th)

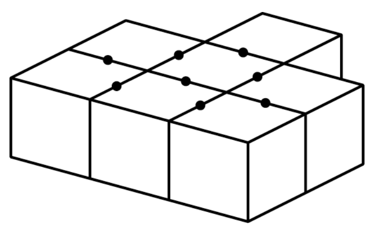
1. 40% of 150 = 0.4 x 150 = 60.

2. 2^2 x (3^2 + 5) = 4 x (14) = 56.

3. √36 + √25 = 6 + 5 = 11.

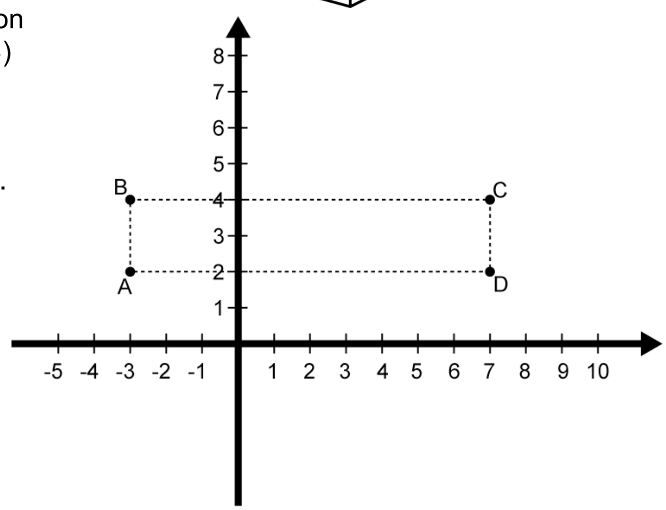
4. There are (1, 2, 3, … 999) 999 natural numbers between 0 and 1 000. The multiples of 4 between 0 and 1 000 are 4, 8, 12, 16, … 996. Altogether, there are (4 = **1** x 4, 8 = **2** x 4, 12 = **3** x 4, … **249** x 4 = 996) 249 multiples of 4 between 0 and 1 000. The probability of randomly choosing a number which is a multiple of 4 is (249/1 000) 83/333.

5. The product of (-1) x 2 x (-3) x 4 x (-5) = -120.



6. Each dot in the diagram accounts for 2 glued faces. In all, there are (8 x 2) 16 faces that have glue on them.

7. The value of the denominator in the equation 1/6 + 1/12 = 1/? Is (2/12 + 1/12 = 3/12 =1/4) 4.

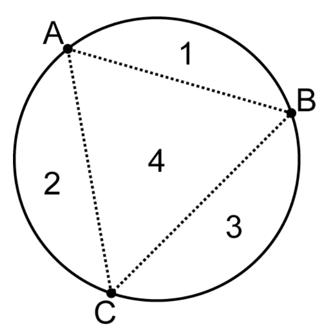


8. When points A (-3 , 2), B (-3 , 4), C (7 , 4), and D (7 , 2) are joined, we get a rectangle.

9. A speed of 10 m/s is equivalent to a speed of (10 m/s = 36 000 m/h) 36 km/h.

10. If n x 6 = 50, then n = 50/6 and n x 21 is equal to (21 x 50/6) 175.

11. The surface of the circle is divided into 4 regions.



12. If 4 eggs are used for 480 grams of flour, this represents (480 ÷ 4) 120 g per egg. For 720 g, we must use (720 ÷ 120) 6 eggs.

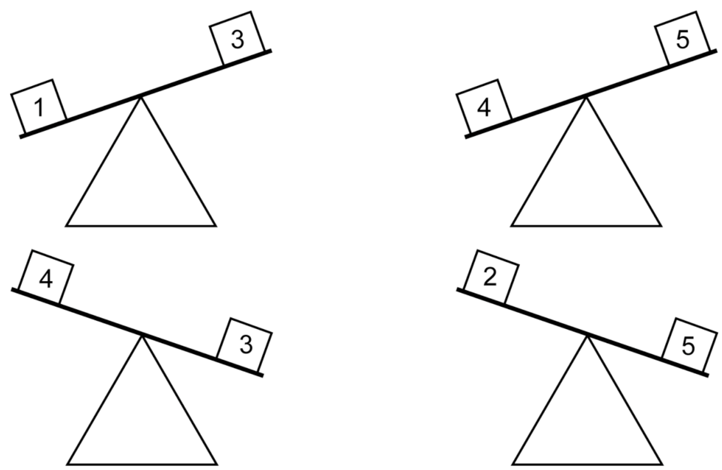
13. Between 30 and 40, there are (31, 37) 2 prime numbers.

14. If the system is in a state of equilibrium, this means that the weight that pushes down on one side is equal to the weight that pushes down on the other. We can write the equation: X - 100 = 75. This equation yields X = 175 kg.

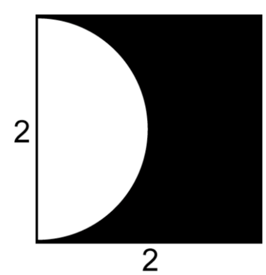
15. The value of $12^3 \times 13^3 = (12 \times 12 \times 12) \times (13 \times 13 \times 13) = (12 \times 13) \times (12 \times 13) \times (12 \times 13) = 156^3$.

16. The square of $\sqrt{5}$ is equal to $(\sqrt{5})^2 = 5$.

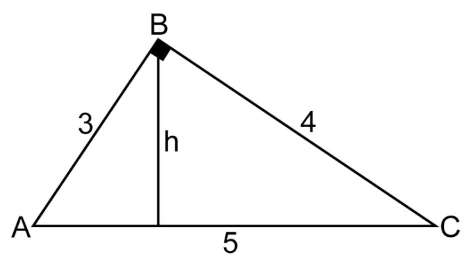
17. Judging from the diagram below, box 1 is heavier than box 3, box 3 is heavier than box 4, box 4 is heavier than box 5, and box 5 is heavier than box 2. The box which is the lightest is box 2.



18. The area of the half-circle is $(\pi\ (1)^2 \div 2)\ \pi/2$. The area of the square is 4. The area of the shaded region is $4 - \pi/2$.



19. The area of triangle ABC is $(3 \times 4 \div 2)$ 6. The value of AC can be found by using $AC^2 = 3^2 + 4^2$. We find AC = 5. The area of triangle ABC is also given by $5 \times h \div 2 = 6$. This equation yields the value of h (the height of triangle ABC). This value is 2.4.

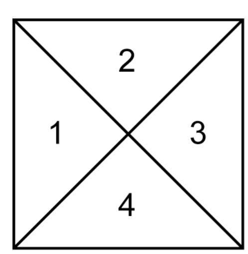


20. The volume of a right cylinder is given by $\pi \times r^2 \times h$. The volume of this cylinder is $(\pi \times 5^2 \times 20)$ 500π cm^3.

21. I can climb up a flight of 3 stairs in 3 different ways (1 - 1 - 1, 1 - 2, 2 - 1).

22. $(\sqrt{2} + 1)^2 = (\sqrt{2} + 1) \times (\sqrt{2} + 1) = 2 + 2\sqrt{2} + 1 = 3 + 2\sqrt{2}$.

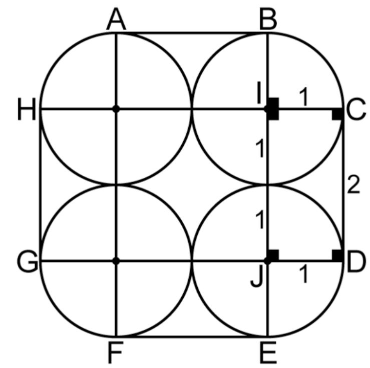
23. When Melissa writes the natural numbers from 1 to 9, she writes **9** digits. From 10 to 19, she writes (10 x 2) **20** digits. From 20 to 49, she writes (3 x 20) **60** digits. From 50 to 54, she writes **10** digits. The 100th digit written is a 5.



24. The area of the square is (4×4) 16 cm^2. The area of one triangle is $(16 \div 4)$ 4 cm^2.

25. The expression $1^n + 2^n$ is divisible by 3 for n = 1, because $1^1 + 2^1 = 3$. It is not divisible by 3 for n = 2, because $1^2 + 2^2 = 5$. It is divisible by 3 for n =3, because $1^3 + 2^3 = 9$. This expression is divisible by 3 for every n which is an odd number.

26. In the diagram opposite, we have 4 congruent circles (with radii equal to 1) that are tangent to each other. The line ABCDEFGH that circumscribes the 4 circles is either straight (line segments AB, CD, EF, and GH) or curved (arcs BC, DE, FG, and HA). Line segment CD is really a common tangent to the circles I and J. Points C and D are two points of contact. We know that a tangent to a circle is perpendicular to the radius at the point of contact. Angles ICD and CDJ are right angles. Line segments IC and ID are parallel and congruent (2 radii of the same circle). Quadrilateral ICDJ is a rectangle. Line segment CD is equal to 2 . Line segments AB, EF, and GH are also equal to 2. Angle BIC is also a right angle; it is the supplementary angle of angle CIJ. Arc BC is equal to one quarter of a circle (the central angle is equal to 90°). Arcs DE, FG, and HA are all quarter-circles. Together, these four arcs are equal to (2 x π x 1) 2π. Together, line segments CD, FE, GH, and AB are equal to (4 x 2) 8. The length of the line that circumscribes the 4 circles is 2 π + 8.



27. The 3 prime numbers between 25 and 40 are 29, 31, and 37. We must reject 29, because it is not consistent with the premises of the problem. Andrea being the oldest, we can infer that she was born when Mathusalem was (31 + 1) 32 years old. Melissa was born 4 years later: this means that Mathusalem was (32 + 4) 36 years old, which is one less than the other prime number (37). Andrea was born when Mathusalem was 32 years old.

28. 120% = 120/100 = 12/10 = 6/5.

29. If x represents the tens digit and y the units digit, the number that we are seeking is equal to 10 x + y. We can write that 10 x + y = 7 (x + y). This equation yields x = 2 y. If y = 1, x = 2, if y = 2, x = 4, if y = 3, x = 6, and if y = 4, x = 8. There are 4 numbers that are equal to 7 times the sum of their digits. The largest is 84. The product of its digits is equal to (8 x 4) 32.

30. If n is a natural number greater than 1 and if n! = 1 x 2 x 3 x 4 x … x n, then the value of 4! is (1 x 2 x 3 x 4) 24.