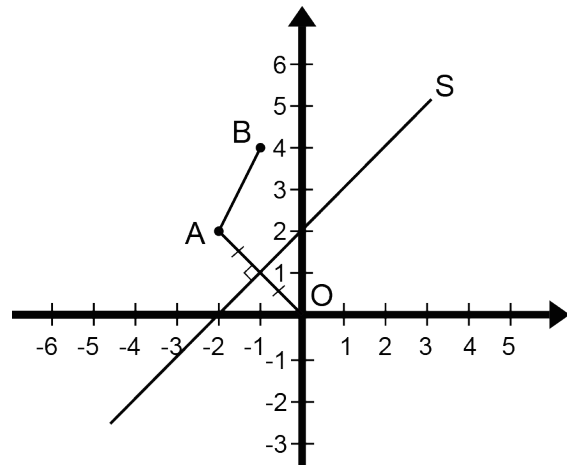


# Mathematica Centrum

Together, let's shape the mathematicians of the future

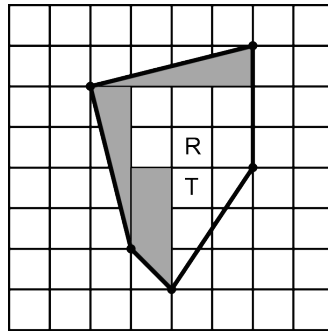
## EULER PREPARATORY TEST 2014 DETAILED SOLUTIONS

1. The value of  $n$  in the equation:  $n \times 5\% = 100$  is  $(100 \div 0,05) = 2\,000$ .
2.  $\frac{3}{4}$  of  $\frac{1}{4}$  of  $16 = \frac{3}{16} \times 16 = 3$ .
3. The value of  $(-2 - 5) + (-5 - 3)$  is  $(-7 + -8 = -7 - 8) = -15$ .
4.  $2 \times 3 - 5 \times -3 = 6 + 15 = 21$ .
5.  $(\frac{1}{6} - \frac{1}{3}) \times \frac{1}{5} = (\frac{1}{6} - \frac{2}{6}) \times \frac{1}{5} = -\frac{1}{6} \times \frac{1}{5} = -\frac{1}{30}$ .
6. The product of 4 prime numbers is never smaller than  $(2 \times 3 \times 5 \times 7) = 210$ .
7. The sum of all natural numbers less than 49 that are square numbers is  $(1 + 4 + 9 + 16 + 25 + 36) = 91$ .
8. A speed of 60 km/h is closest to  $(60 \times 1\,000 \text{ m} \div 3\,600 \text{ s} = 16,666 \dots) = 17 \text{ m/s}$ .
9. The product 110 can only be written as  $2 \times 5 \times 11$ . Matusalem could not have chosen 4 different integers between 1 and 20. The problem is impossible.
10. It takes 6 minutes to fill  $\frac{3}{7}$  of a bath-tub. At this rate, the number of extra minutes needed to fill another seventh ( $\frac{1}{7}$ ) is  $(6 \div 3) = 2$  minutes.
11. One quarter of 3 hours and 20 minutes is equal to  $((3 \times 60 + 20) \div 4) = 50$  minutes.
12. Number 21 has only 4 factors (1, 3, 7, and 21). The number 16 has five (1, 2, 4, 8, and 16). All the other numbers have more than 5 factors.
13. The image of point A of line segment AB, if S is a flip line, is at O(0, 0). Line segment AO is perpendicular to S (see diagram opposite).

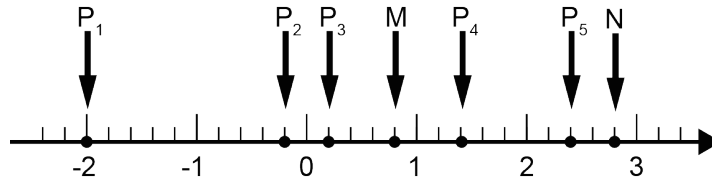


14. The smallest positive integer that, when multiplied by 12 will yield a square number is  $(12 \times 3 = 36)$  3.

15. The area of the pentagon opposite is equal to the sum of the areas of the two shaded triangles, of rectangle R, of the shaded trapezium, and of triangle T. This sum is equal to  $(2 \text{ cm}^2 + 2\text{cm}^2 + 6 \text{ cm}^2 + 2.5 \text{ cm}^2 + 3 \text{ cm}^2)$   $15.5 \text{ cm}^2$ .



15. The point representing the average of  $P_1$  and  $P_5$  is  $P_3$  because it is equal to  $((-2 + 2.4) \div 2)$   $0.2$ .



17. Number 7 is a happy number because  $7^2 = 49$ ,  $4^2 + 9^2 = 97$ ,  $9^2 + 7^2 = 130$ ,  $1^2 + 3^2 + 0^2 = 10$ , and  $1^2 + 0^2 = 1$ .

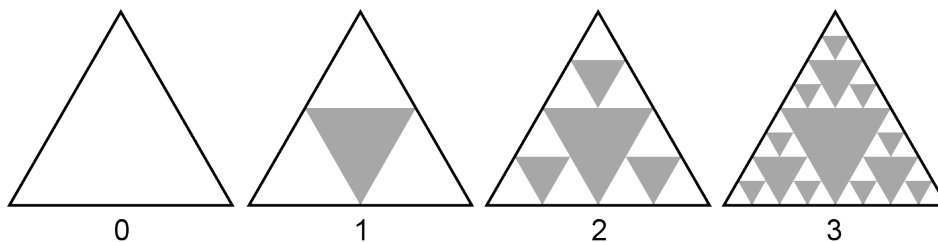
18. The area of the rectangle is equal to the area of a square. The side of the square and the base of the rectangle are doubled. If  $c$  represents the side of the initial square, the area of the new square is  $((2c)^2) 4c^2$ , that of the new rectangle is  $(2 \times c^2) 2c^2$ . The difference between the area of the new square and that of the new rectangle is  $(4c^2 - 2c^2) 2c^2$ .

19. Since  $W$  is not sitting beside  $X$  nor  $Y$ , it must be sitting at one of the two ends. Let's place him on the left end of the bench.  $Z$  must be sitting just to the right of  $W$  because  $X$  and  $Y$  cannot be sitting beside  $W$ .  $Y$  must be sitting just to the right of  $Z$  and  $X$  is sitting on the right end of the bench. Today,  $Z$  and  $Y$  are sitting between the other two.



20. If  $x = -3$ , the value of  $x + x^2 + x^3$  is equal to  $(-3 + (-3)^2 + (-3)^3)$   $-21$ .

21. On step 1,  $(3^0)$  one triangle is removed, on step 2  $(3^1)$  3 triangles are removed, on step 3  $(3^2)$  9 triangles are removed. The number of triangles that will be removed on the fifth step (step 4) is equal to  $(3^3)$  27.



22. The total area of this prism is  $(2 \times 36 \text{ cm}^2 + 2 \times 24 \text{ cm}^2 + 2 \times 24 \text{ cm}^2)$   $168 \text{ cm}^2$ . Its volume is  $(36 \text{ cm}^2 \times 4 \text{ cm})$   $144 \text{ cm}^3$ . The ratio (in  $\text{cm}^2$  by  $\text{cm}^3$ ) of the total area compared to the volume is  $(168 \text{ cm}^2 \div 144 \text{ cm}^3)$   $7/6 \text{ cm}^2/\text{cm}^3$ .

23. If  $M_1 = 12$ ,  $N_1 = 8$ ,  $M_2 = 16$ , and  $N_2 = 24$ , then  $P_1 = 24$ ,  $G_1 = 4$ ,  $P_2 = 48$ , and  $G_2 = 8$ . The product  $P_1 \times P_2 \times G_1 \times G_2$  is equal to  $(24 \times 48 \times 4 \times 8)$   $36864$ .

24. If  $n$  is a positive integer and  $n^2 + 3$  is odd, then  $n^2$  is even and consequently  $n$  is even. The expression  $n^3 - n$  is always even.

25. The sum of  $n$  positive integers is equal to 8. If  $P$  represents the product of these  $n$  numbers, the greatest possible value of  $P$  is  $(3 \times 3 \times 2)$   $18$ .

26. The factors of 12 are 1, 2, 3, 4, 6, and 12. The number of even factors of 12 is equal to 4.

27. If  $A \times B = 12$ ,  $B \times C = 20$ , and  $C \times D = 40$ , we can write that  $\frac{A \times B}{B} \times \frac{B \times C}{C} \times \frac{C \times D}{D} = 9600$ . This equation can be written as  $A \times D \times (BC^2) = 9600$ , from which, we find the value of  $A \times D$  which is equal to 24.

28. If  $n$  is a positive integer, the number of terms of this sequence that are even and less than 100 is  $(4, 10, 16, \dots, 94)$   $16$ .

